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The Boundedness of Euclidean Gravity and the Wavefunction of the Universe

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ABSTRACT: When the semi-positive cosmological constant is dynamical, the naive Euclidean Einstein action is unbounded from below and the Hartle-Hawking wavefunction of the universe is not normalizable. With the inclusion of back-reaction (a crucial point), the presence of the metric perturbative modes (as well as matter fields) as a radiation term is introduced by quantum fluctuation. They act as the environment (that is, to be integrated or traced out), and introduce a correction term that provides a bound to the Euclidean action. As a result, the improved wavefunction is normalizable. That is, decoherence plays an essential role in the consistency of quantum gravity. In the spontaneous creation of the universe, this improved wavefunction allows one to compare the tunneling probabilities from absolute nothing (i.e., not even classical spacetime) to various vacua (with different large spatial dimensions and different low energy spectra) in the stringy cosmic landscape.

Contents

1. Introduction

In the probing of the origin of our universe, a particularly attractive idea is the tunneling from absolute nothing (here, nothing means not even classical spacetime) [1], or equivalently, the Hartle-Hawking (HH) no-boundary wave function of the universe [2]. The basic idea is illustrated in Figure 1. It is strongly believed that, in superstring theory, there is a vast number of stable and meta-stable vacua, with up to 9 or 10 large spatial dimensions. This is the cosmic landscape. If we can reliably calculate the tunneling probability from absolute nothing to any point in this vast landscape, one may argue that the origin of our universe should be the point in the landscape with the largest tunneling probability. This tunneling probability is given by (the absolute square of) the wavefunction of the universe. However, the HH wavefunction is not normalizable. Furthermore, it gives an answer that
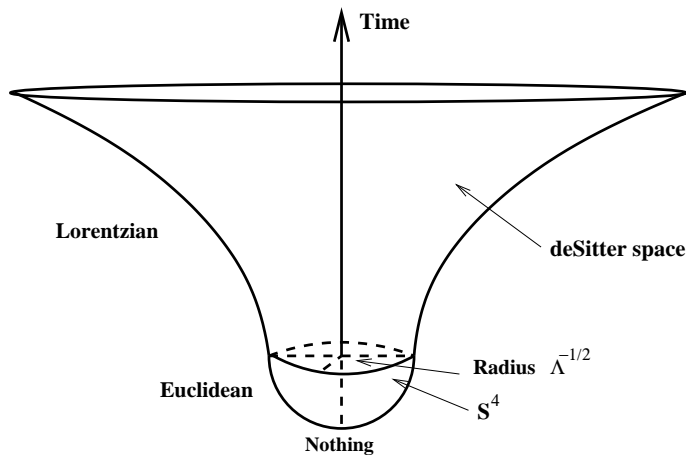
Figure 1: Tunneling from absolute nothing to a deSitter or Inflationary Universe.

contradicts observations. It is well-known that the problem lies in the infrared/macroscopic limit, so quantum/stringy corrections will not be helpful. In Ref [3], we conjectured that a normalizable wavefunction results if decoherence effect is included. One can then apply this improved wavefunction to find the preferred vacuum in the cosmic landscape (the one emerging from tunneling from nothing with the largest tunneling probability), which then evolves to today's universe. We call this proposal the Selection of the Original Universe Principle (SOUP); that is, today's universe must lie along the road that starts with the original preferred vacuum, arguably the 4-dimensional inflationary universe supported by observations. In this paper, we investigate more carefully the decoherence effect on the tunneling probability/wavefunction. As expected, the result supports the basic underpinning of SOUP, though the details are somewhat more involved.

Consider the simple Einstein theory in 4 dimensions with a positive cosmological constant :

$$S_E = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} \left(R - 2\Lambda\right) \tag{1.1}$$

The tunneling probability from nothing to a closed deSitter (or inflationary) universe with cosmological constant Λ is given by

$$\Gamma = |\Psi|^2 \simeq e^{-S_E} \qquad S_E = -\frac{3\pi}{G\Lambda} \tag{1.2}$$

where S_E is the Euclidean action of the S^4 instanton [5]. Suppose Λ is dynamical, as in a model with 4-form field strengths [6]. The tunneling probability Γ seems to allow us to pick out the universe with the largest probability. However, the Euclidean action S_E is unbounded from below as $\Lambda \to 0$, so $\Gamma \to \infty$. This has at least 2 obvious problems :

(1) The wavefunction $\Psi \simeq e^{3\pi/2G\Lambda}$ is not normalizable. Furthermore, one can easily show that there are other topological instantons with even more negative Euclidean action and so larger (actually, infinitely larger) tunneling probability [7]. This implies the presence of an inconsistency.

(2) Phenomenologically, since $\Gamma \to \infty$ as $\Lambda \to 0$, it will imply the preference of tunneling to a flat universe with zero Λ, which contradicts the big bang history of our universe. Since the size of the universe, the cosmic scale factor $a, \sim 1/\sqrt{\Lambda}$, this will imply that the tunneling is to a universe of super-macroscopic size, contradicting one's intuition that tunneling is a quantum process and so should be microscopic.

This issue is an outstanding problem since the early 1980s. The inconsistency has prevented the proper application of the whole idea. Possible resolution to this problem has been suggested (see [4] and references therein):

- Instead of the usual $t \to -i\tau$, one may choose instead to rotate time to Euclidean time via $t \to i\tau$ [8]. This may work for pure gravity, but the inclusion of a scalar field (e.g., as required for inflation) leads to catastrophic consequences, since the scalar field theory becomes unbounded from below [9].

- One may argue that this problem will be corrected by quantum corrections or string theory corrections. However, it is easy to see that this is unlikely to be the case. Note that the problem occurs for small Λ, or large universe, since the cosmic scale factor $a \sim 1/\sqrt{\Lambda}$. So this is more like an infrared or macroscopic problem than an ultra-violet problem. In fact, one can easily see that the loop correction [10, 11] does not solve the problem. Also, recent work [12], where the exact HH wavefunction is obtained in topological string theory, it seems that $\Lambda \to 0$ is again preferred, just like the original HH wavefunction.

- This last property, namely, that the unboundedness problem becomes acute when the deSitter universe becomes large, or macroscopic (actually super-macroscopic), naturally suggests that the resolution should lie in decoherence [3].

In Ref. [3], we argue that the mini-superspace formulation is inadequate, and we propose that the inclusion of decoherence effects due to other modes provides a lower bound to S_E. We then speculate how the improved wavefunction may be used to select the stringy vacuum with the largest tunneling probability from absolute nothing. In this paper, we shall show that decoherence indeed provides a bound to the Euclidean gravity action, though the formula for the tunneling probability may be more involved. Here, we shall focus on the pure 4-D Einstein gravity case. Generalization to more dimensions is straightforward and will be briefly discussed.

The basic idea of the approach is widely used in physics. Given any complicated problem, we usually follow only a limited set of degrees of freedom, called the system. The remaining degrees of freedom, called the environment, are either ignored, or, in a better approximation, integrated out. In decoherence, integrating out the environment can cause the quantum system to behave like a classical system. In effective field theory in particle physics, the massive modes are integrated out to produce higher dimensional operators (interaction terms) for the light modes. A famous example is the integrating out of the W and Z bosons in the electroweak theory that yields the 4-Fermi weak interactions. Another example is the integrating out of the hidden sector in supergravity phenomenology. (Note that this has nothing to do with loop corrections.) In the Wilson approach in quantum
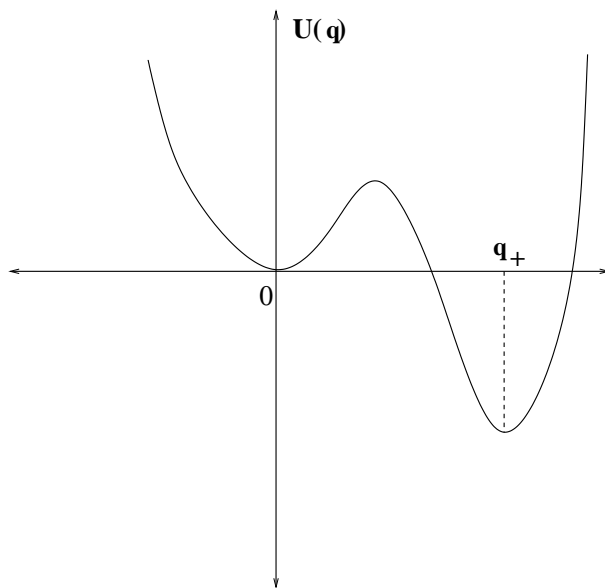
Figure 2: Potential $V(q)$ with a metastable minimum at $q = 0$, and a stable minimum at $q = q_+$.

field theory, where high momentum modes are integrated out, and in the above cases, it is clear that the physics may crucially depend on the effects coming from integrating out the unobserved modes; that is, they cannot be ignored. Of course, loop corrections can also induce dynamics/interactions that are not present in the tree level, as for example in the Coleman-Weinberg model, in light-light scattering, and in the running of couplings. In QCD, the running of the coupling emerges from the renormalization group improved quantum corrections, not just a normal 1-loop effect. We argue that this is also the situation in the study of the wavefunction of the universe. One may consider our result as a back-reaction improved quantum correction, not just a normal 1-loop effect.

The basic idea applied to tunneling is quite simple. It is well-known that the quantum tunneling of a particle with mass M, or the system, is suppressed if it interacts with an environment. Consider a particle at $q = 0$ in the potential $V(q)$ as shown in Figure 2. In the WKB approximation, its tunneling rate is given by

$$\Gamma \simeq \exp(-S_0)$$
$$S_0 = \int_0^{q_0} \sqrt{2MV(q)}dq \tag{1.3}$$

where S_0 is the Euclidean action of the bounce, i.e., the instanton solution [13]. Note that, for $V(q)$ bounded from below, S_0 is bounded from below, as required by consistency.

In a more realistic situation, the particle interacts with a set of other particles, or modes, say x_α. However, we are only interested in the quantum status of q, so these other modes are integrated out in the path integral, or traced over in the density matrix formulation. They provide the environment. Their presence typically introduces a frictional force to the evolution of q. It was shown by Sethna [14] and by Caldeira and Leggett [15],

that the bounce S_0 increases to $S \simeq S_0 (1 + \hat{\eta})$ where $\hat{\eta} > 0$ is proportional to the coefficient of friction (see Appendix A for details). That is, the interaction with the environment suppresses the tunneling rate. (This suppression takes place even if no friction is generated.) One may understand this result in a number of (equivalent) ways :

- As a quantum system,

$$S = \int \sqrt{2MV(q, x_\alpha)} ds$$
$$ds^2 = dq^2 + \sum \frac{m_\alpha}{M} dx_\alpha^2 \tag{1.4}$$

That is, the increase in S is due entirely to the longer path length in the many dimensional (q, x_α) space.

- The interaction of q with the x_α interferes with its attempt in tunneling. One may view the interaction with x_α as attempts to observe q. Repeated measurements of q or repeated attempts of measuring q suppresses the tunneling rate. This is analogous to the Zeno or Watch Pot effect.

- The interaction of q with the environment x_α diminishes the quantum coherence. As a consequence, the system behaves more like a classical system than like a quantum system. Since tunneling is a pure quantum phenomenon, it should be suppressed as the system becomes more macroscopic/classical. We shall refer to this as decoherence, by which we mean the process where a quantum system behaves more classically (i.e., less quantum) via its interactions with the environment [16].

Here, we study this effect in quantum gravity, in the tunneling from nothing scenario. In this case, the cosmic scale factor a plays the role of the system, while the metric fluctuations around a (and any matter field modes) play the role of x_α, i.e., the environment. Figure 3 illustrates this situation. Since we measure only a, the metric fluctuations are integrated out in the path integral (or traced out in the density matrix). As expected, we shall show that their presence suppresses the tunneling from nothing to a deSitter universe. As expected, the decoherence effect is negligible for large cosmological constant (small size universe), but becomes increasingly important as $\Lambda \to 0$, as it suppresses the tunneling probability when $a = 1/\sqrt{\Lambda/3}$ becomes macroscopic. In contrast to a normal quantum system, the inclusion of the environment is of fundamental importance in quantum gravity, since the corrected Euclidean action is now bounded from below.

The coupling of the metric fluctuations and scalar fields to $a(\tau)$ are more complicated than that of the above quantum system. Each metric perturbative mode behaves like a simple harmonic oscillator but with time-dependent (or a-dependent) mass and frequency. However, the real subtlety of the calculation comes in another way. If we treat the metric fluctuation modes as pure perturbations, we shall get nothing except the loop correction, a known result in Euclidean gravity. This is not hard to see. The Euclidean action in minisuperspace (that is, keeping only the cosmic scale factor) is given by

$$S_E = \frac{1}{2} \int d\tau \left(-a\dot{a}^2 - a + \lambda a^3 \right) \tag{1.5}$$
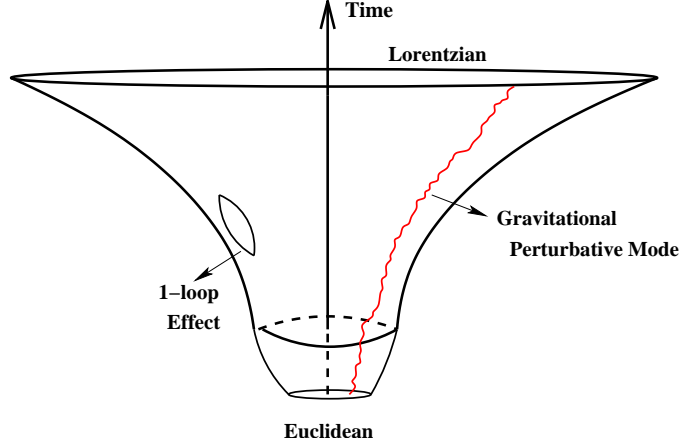
Figure 3: Typical tunneling involves metric and matter excitations as well. Loop effects may also be included, though they are not important for our discussions.

where a rescaling has rendered τ, $a(\tau)$, the Hubble constant $H = 1/\sqrt{\lambda}$ and $\lambda = 2G\Lambda/9\pi$ dimensionless. For S^4, $Ha(\tau) = \sin(H\tau)$, where south pole (north pole) corresponds to $H\tau = 0$ (π). This path gives the Euclidean action (1.2). For a fluctuation mode f that satisfies the classical equation, the Euclidean classical action can be written as a surface term $S_E \simeq a^3 f\dot{f}|_0^{\pi/H} = 0$, since $a(\tau) = 0$ at the two poles. As a result, no decoherence term is generated. (Including an additional boundary term makes no difference.) However, we find that the back-reaction is crucial for getting the correct answer. Instead of using the unperturbed $a(\tau)$ given above for the S^4 geometry, we leave it arbitrary during the tracing out of the fluctuation modes. In the path-integral formalism, one starts with the path integral that includes the scale factor a as well as the perturbations around a. The following, for example, shows the inclusion of the metric tensor perturbations t_n

$$Z = \int D[a] \prod_n \int_{t_n^i}^{t_n^f} D[t_n] e^{-S_E} \exp\left(-\frac{1}{2} \int_{-T/2}^{T/2} d\tau a^3 \sum_n [\dot{t_n}^2 + \omega_n(a)^2 t_n^2]\right) \qquad (1.6)$$

Tracing out the perturbations, we have

$$Tr[Z] = \prod_n \int dt_n^i \int dt_n^f \delta(t_n^i - t_n^f) \; Z \qquad (1.7)$$

This results in a new term in the modified action

$$S_{E,dC} \simeq \frac{1}{2} \int d\tau \left(-a\dot{a}^2 - a + \lambda a^3 + \frac{\nu}{\lambda^2 a}\right) \qquad (1.8)$$

where the last term, coming from integrating out the perturbative modes, behaves like ordinary radiation. ν is a constant that measures the number of perturbative modes. We then solve for $a(\tau)$ and obtain the corrected Euclidean action $S_{E,dC}$ in the saddle-point approximation. It turns out that the ν term modifies the shape of the S^4 instanton to barrel-like, as shown in Figure 4. Since the effect is perturbative in nature, we expect the
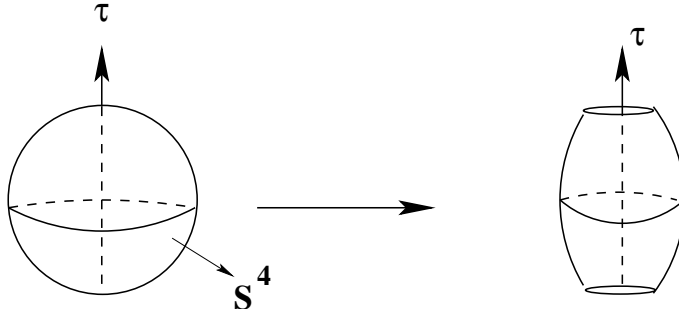
Figure 4: Modification of the S^4 instanton due to the presence of decoherence, i.e., the inclusion of the environment.

barrel to have the same topology as S^4. We see that, due to the back-reaction, $a(\tau)$ does not vanish at the two ends. However, the contribution of the end plates of the barrel to $S_{E,dC}$ happens to be zero.

After tracing over the metric fluctuations in this way, the rate of tunneling from nothing (i.e., no classical spacetime) to a deSitter universe (much like the inflationary universe) is now given by $\Gamma \simeq \exp(F) = \exp(-S_{E,dC})$, where

$$-F = S_{E,dC} = S_{E,0} + D \simeq -\frac{3\pi}{G\Lambda} + c\left(\frac{3\pi}{G\Lambda}\right)^2 \tag{1.9}$$

where D is the decoherence term and c depends on the cut-off. In string theory, that cut-off is naturally provided by the string scale. Note that $S_{E,dC}$ is now bounded from below. See Figure 5. Here, c in string theory also depends on the string spectrum, so c should be calculated for each vacuum. We find that

$$c = \frac{3}{2}n_{dof}\left(\frac{2G}{3\pi l_s^2}\right)^2 \tag{1.10}$$

where l_s is the string scale ($\alpha' = (l_s/2\pi)^2$) and n_{dof} is the number of light degrees of freedom included in the environment. For the pure gravity case, we have $n_{dof} = 2$ for the two tensor modes. For small Λ, below a critical value, the tunneling is actually totally suppressed.

This is quite understandable. A precise S^4 spherical geometry leads to the pure de Sitter space, which by definition excludes any radiation. If we do not allow back-reaction, then the system $a(\tau)$ actually cannot feel the presence of the environemnt. Allowing back-reaction, the quantum fluctuation during the spontaneous creation of the universe generates some radiation (even though no radiation is introduced classically). They act as the environment. The presence of this environment modifies the geometry in a way so that tunneling is to a universe with both a cosmological constant and some radiation, with a suppressed tunneling rate. The presence of the radiation generates the decoherence term. Note that, in integrating out the perturbative modes, the case with zero amplitude for the perturbative modes is also included. As we see, the amount of radiation is proportional to $1/\Lambda^2$, so there is more radiation in a larger universe.
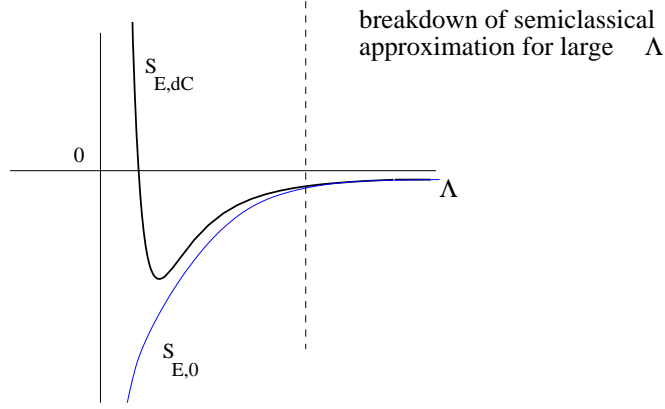
Figure 5: $S_{E,dC}$ and $S_{E,0}$ as functions of the cosmological constant Λ. We see that, with the inclusion of the decoherence effect, the Euclidean action for the S^4 instanton is now bounded from below. For large Λ, the semi-classical approximation breaks down. For small Λ, tunneling is actually completely suppressed.

What we consider fundamental is that the decoherence effect actually provides the Euclidean action S_E of pure gravity with a lower bound. Since the metric fluctuation contribution to S_E cannot be turned off, they must be included in the evaluation of the tunneling rate. For usual quantum system, S_0 is bounded from below. So one may view this friction/environment/decoherence effect as a correction, albeit it may be very big. In quantum gravity, this effect resolves the unboundedness problem. That the quantum fluctuation provides a natural source to cure the boundedness problem implies that quantum gravity is actually self-consistent in the macroscopic regime.

This effect renders the system more macroscopic and so less quantum. That is, as a becomes large, its interaction with the metric fluctuations and matter fields should suppress the tunneling rate. Indeed, we show that the inclusion of the metric fluctuation decreases the tunneling rate, as expected. Note that the decoherence term is not the usual perturbative quantum correction. For large Λ, where quantum correction is expected to be large, the decoherence term actually becomes negligible. One can include the quantum corrections; however, they do not change the qualitative behavior for moderate values of Λ, i.e., $G\Lambda << 1$.

Let us gain some idea of the magnitude of F. Following Eq.(1.9), we find the value of Λ with maximum tunneling probability is

$$\Lambda_{max} = \frac{4 n_{dof} G}{\pi l_s^4} \rightarrow F_{max} = \frac{3}{2 n_{dof}} (2\pi M_{Pl} l_s)^4 \qquad (1.11)$$

For the string scale a few orders of magnitude smaller than M_{Pl}, we can easily have $F \simeq 10^{10}$ or larger. On the other hand, the critical value of Λ is $\Lambda_c = \Lambda_{max}/2$. For $\Lambda < \Lambda_c$, the barrel-shaped instanton is destroyed. At Λ_c, $F(\Lambda_c) = 0$, so the tunneling probability at Λ close to Λ_c is already negligibly small compared to that at Λ_{max}.

Having resolved the outstanding problem mentioned above, one may then apply this consistent tunneling approach to select a preferred vacuum in the cosmic landscape in string theory. It is straightforward to generalize Eq.(1.9) to arbitrary spatial dimensions. In particular, for ten dimensional spacetime, we have

$$- F = S_{E,dC} \simeq S_{E,10} + c \left(\frac{V_{10}}{l_s^{10}} \right) \tag{1.12}$$

where $S_{E,10}$ is the 10-D Euclidean action determined in mini-superspace and V_{10} is the 10-dimensional volume of the instanton. Note that (see Ref [3]) $S_{E,10}$ reduces to $-3\pi/G\Lambda$ for a vacuum where the extra 6 dimensions are compactified. For each vacuum, c depends on the spectrum. It may also depend on the compactification and dilaton moduli. Knowing these properties of each vacuum allows one to calculate its tunneling probability from nothing. One may estimate the size of c by comparing to the 4-D case:

$$c \simeq \frac{n_{dof}}{\pi M_{Pl}^2 l_s^2 g_s^2} \tag{1.13}$$

with $8\pi G = 1/M_{Pl}^2$ and g_s is the string coupling. Since the string scale is expected to be a few orders of magnitude below the Planck scale, we expect c to be a small number. In [3], we consider the suppression of tunnelling in the context of the spontaneous creation of the universe. The suppression happens due to the effect of both the gravitational and matter perturbations on the bounce solution to the Euclidean Einstein equations. The general idea, loosely speaking, is to separate the "universe" into a "system" (the pure gravitational bounce) and the "environment" (the perturbations). One is interested in measuring properties of the system and in order to do so one simply traces out the environmental degrees of freedom. The effect of the environment can be significant and such effects have been studied in various systems. In the previous work [3] we estimated this effect on the tunnelling probability of the universe by considering the unperturbed deSitter space as the "system" and the "environment" consists of the metric perturbations and matter fields. We see that the qualitative features are robust, though the details are somewhat more involved. (There we used V_9 instead of V_{10} for Eq.(1.12)). For large Λ, the semi-classical approximation breaks down. For small Λ, additional decoherence effect may be important. (Here we have calculated only the leading order.) Fortunately, the range of Λ where the tunneling probability is largest seem to lie in the region where the approximation is most reliable; and we are interested in vacua with large tunneling probabilities. Since the tunneling probability drops off rapidly as $\Lambda \to 0$ and $\Lambda \to \infty$, their precise values are not as important to us. With some luck, we may use the above formula to locate the preferred set of vacua in the cosmic string landscape. Note that c depends on the spectrum at each point in the landscape.

As one can see in Figure 5, intermediate values of Λ, much like the inflationary universe that describes the history of our universe [17] seems to be preferred. As in Ref [3], we find phenomenologically that 10-dimensional deSitter-like vacua (with $F \simeq 10^9$ for instantons S^{10}, $S^5 \times S^5$, $S^4 \times S^3 \times S^3$) are not preferred while supersymmetric vacua in any dimension have essentially zero tunneling probability. Also, tunneling to vacua very much like our

today's universe (with a very small dark energy) seems to be severely suppressed. Tunneling to a universe with quantum foam seems not preferred either. Among the known vacua, the preferred ones are the 4-D brane inflation [18] as realized in a realistic string model [19], very much like the inflationary universe that our universe has gone through (with $F \sim 10^{16}$). Although the details of the decoherence term obtained here is somewhat different from that used in Ref [3], we see that the qualitative features summarized there remain true.

Decoherence and related issues in quantum/Euclidean gravity have been studied earlier [20–24], where they are mostly concerned with the evolution of the inflationary universe. Here, we study decoherence in the quantum tunneling in gravity. To be self-contained, we shall review some of the relevant formalisms developed there.

In Sec. 2, we review the tunneling to closed deSitter space and perturbative modes around the cosmic scale factor $a(\tau)$. In Sec. 3, we evaluate the contribution due to these modes to the effective Euclidean action. Here we find that no decoherence term is generated in the S^4 background. To see why back-reaction is important, the effect is calculated in the background of a "squashed" S^4 geometry. The reader may choose to skip this section. In Sec. 4, we study the modified bounce including back reaction. We derive the effective Euclidean action after integrating out the metric perturbation with an arbitrary background $a(\tau)$. In Sec. 5, We find the modified bounce solution from the effective Euclidean action. This allows us to obtain the improved wavefunction and tunneling probability from nothing. In Sec. 6, we show that the new term in the improved Euclidean action behaves like ordinary radiation. We see also how the new term appears in the Wheeler-DeWitt equation and how it influences the tunneling amplitude. In Sec. 7, we discuss the implication of the main result and its connection to the proposal of Ref [3]. Sec. 8 contains some discussions and Sec. 9 contains a summary and further remarks. Some of the details are relegated to appendices. For the sake of completeness, a number of known results are reviewed extensively.

2. Setup

2.1 Notations

We shall render various quantities dimensionless for the ease of calculation. The conventions we follow are those of [2]. The Euclidean action is defined as

$$S_E = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} \, (R - 2\Lambda) \tag{2.1}$$

The Euclidean metric is given by

$$ds^2 = \sigma^2 \left(d\tau^2 + a(\tau)^2 d\Omega_3^2 \right) \tag{2.2}$$

where is $\sigma^2 = 2G/3\pi$. With this metric ansatz, the action becomes

$$S_E = \frac{1}{2} \int d\tau \left(-a\dot{a}^2 - a + \lambda a^3 \right) \tag{2.3}$$

where $\lambda = \sigma^2 \Lambda/3 = 2G\Lambda/9\pi$. So λ, τ, and a are all dimensionless.

2.2 The deSitter Space

In this section we give a summary of the result of [20]. Consider a compact three-surface S^3 which divides the four-manifold M into two parts. One can introduce the coordinates x^i ($i = 1, 2, 3$) and a coordinate t such that S^3 is the surface at $t = 0$. The metric takes the form

$$ds^2 = -(N^2 - N_i N^i)dt^2 + 2N_i dx^i dt + h_{ij} dx^i dx^j. \tag{2.4}$$

where N and N_i are the lapse function and the shift vector, respectively. The action is given by

$$S = \int (L_g + L_m) \, d^4 x \tag{2.5}$$

where

$$L_g = \frac{M_P^2}{16\pi} N \left(G^{ijkl} K_{ij} K_{kl} + \sqrt{h} R \right) \tag{2.6}$$

where R is the Ricci scalar of the three-surface and K_{ij} is the second fundamental form given by

$$K_{ij} = \frac{1}{2N} \left(-\frac{\partial h_{ij}}{\partial t} + 2N_{(i|j)} \right) \tag{2.7}$$

In the above expression "|" denotes the covariant derivative. G^{ijkl} is called the metric of the "superspace" and is given by

$$G^{ijkl} = \frac{1}{2} \sqrt{h} \left(h^{ik} h^{jl} + h^{il} h^{jk} - 2 h^{ij} h^{kl} \right) \tag{2.8}$$

In the case of a massive scalar field, the matter Lagrangian L_m is given by

$$L_m = \frac{1}{2} N \sqrt{h} \left(N^{-2} \left(\frac{\partial \Phi}{\partial t} \right)^2 - 2 \frac{N^i}{N^2} \frac{\partial \Phi}{\partial t} \frac{\partial \Phi}{\partial x^i} \right.$$
$$\left. - \left(h^{ij} - \frac{N^i N^j}{N^2} \right) \frac{\partial \Phi}{\partial x^i} \frac{\partial \Phi}{\partial x^j} - m^2 \Phi^2 \right) \tag{2.9}$$

In the Hamiltonian treatment of general relativity one treats h_{ij} and Φ as the canonical coordinates. The canonically conjugate momenta are

$$\pi^{ij} = \frac{\partial L_g}{\partial \dot{h}_{ij}} = -\frac{\sqrt{h} M_P^2}{16\pi} \left(K^{ij} - h^{ij} K \right) \tag{2.10}$$

$$\pi_\Phi = \frac{\partial L_m}{\partial \dot{\Phi}} = N^{-1} \sqrt{h} \left(\dot{\Phi} - N^i \frac{\partial \Phi}{\partial x^i} \right)$$

The Hamiltonian is

$$H = \int d^3 x \left(\pi^{ij} \dot{h}_{ij} + \pi_\Phi \dot{\Phi} - L_g - L_m \right) \tag{2.11}$$

$$= \int d^3 x \left(N H_0 + N_i H^i \right)$$

where

$$H_0 = 16\pi M_P^{-2} G_{ijkl} \pi^{ij} \pi^{kl} - \frac{M_P^2}{16\pi} \sqrt{h} R + \tag{2.12}$$

$$\frac{1}{2}\sqrt{h}\left(\frac{\pi_\Phi^2}{h} + h^{ij}\frac{\partial\Phi}{\partial x^i}\frac{\partial\Phi}{\partial x^j} + m^2\Phi^2\right)$$

$$H^i = -2\pi^{ij}_{|j} + h^{ij}\frac{\partial\Phi}{\partial x^j}\pi_\Phi \tag{2.13}$$

and

$$G_{ijkl} = \frac{1}{2}h^{-1/2}\left(h_{ik}h_{jl} + h_{il}h_{jk} - h_{ij}h_{kl}\right) \tag{2.14}$$

The quantities N and N_i are regarded as Lagrange multipliers. Thus the solution obeys the momentum constraint

$$H^i = 0 \tag{2.15}$$

and the Hamiltonian constraint

$$H_0 = 0. \tag{2.16}$$

2.3 The Perturbations

Now we study the perturbations around the deSitter spacetime. The perturbed deSitter has a three-metric h_{ij} of the form

$$h_{ij} = a^2\left(\Omega_{ij} + \epsilon_{ij}\right) \tag{2.17}$$

where Ω_{ij} is the metric on the unit three-sphere and ϵ_{ij} is a perturbation on this metric and can be expanded in harmonics:

$$\epsilon_{ij} = \sum_{n,l,m}\left[\sqrt{6}a_{nlm}\frac{1}{3}\Omega_{ij}Q^n_{lm} + \sqrt{6}b_{nlm}(P_{ij})^n_{lm} + \right.$$

$$\left.\sqrt{2}c^0_{nlm}(S^0_{ij})^n_{lm} + \sqrt{2}c^e_{nlm}(S^e_{ij})^n_{lm} + 2t^0_{nlm}(G^0_{ij})^n_{lm} + 2t^e_{nlm}(G^e_{ij})^n_{lm}\right] \tag{2.18}$$

where the coefficients a_{nlm}, b_{nlm}, c^0_{nlm}, c^e_{nlm}, t^0_{nlm} and t^e_{nlm} are functions of time but not the three space coordinates. The $Q(x^i)$ are the scalar harmonics on the three-sphere. The $P_{ij}(x^i)$ are given by

$$P_{ij} = \frac{1}{n^2 - 1}Q_{|ij} + \frac{1}{3}\Omega_{ij}Q \tag{2.19}$$

where we have suppressed the n, l, m indices. The S_{ij} are given by

$$S_{ij} = S_{i|j} + S_{j|i} \tag{2.20}$$

where S_i are the transverse vector harmonics. The G_{ij} are the transverse traceless tensor harmonics. Further details can be found in [20, 25]. The lapse, shift, and the scalar field $\Phi(x^i, t)$ can be expanded in terms of harmonics as

$$N = N_0 \left[1 + \frac{1}{\sqrt{6}} \sum_{n,l,m} g_{nlm} Q^n_{lm} \right] \tag{2.21}$$

$$N_i = a(t) \sum_{n,l,m} \left[\frac{1}{\sqrt{6}} k_{nlm} (P_i)^n_{lm} + \sqrt{2} j_{nlm} (S_i)^n_{lm} \right]$$

$$\Phi = \sigma^{-1} \left[\frac{1}{\sqrt{2}\pi} \phi(t) + \sum_{n,l,m} f_{nlm} Q^n_{lm} \right]$$

where $P_i = \frac{1}{n^2-1} Q_{|i}$. The perturbed action is now given by

$$S = S_0(a, \phi, N_0) + \sum_n S_n \tag{2.22}$$

where we have denoted the labels n, l, m, o, and e by the single label n. S_0 is the action of the unperturbed deSitter,

$$S_0 = -\frac{1}{2} \int dt N_0 a^3 \left(\frac{\dot{a}^2}{N_o^2 a^2} - \frac{1}{a^2} - \frac{\dot{\phi}^2}{N_0^2} + m^2 \phi^2 + \lambda \right) \tag{2.23}$$

S_n is quadratic in perturbations and is given by

$$S_n = \int dt (L^n_g + L^n_m) \tag{2.24}$$

where L^n_g and L^n_m are the nth mode gravitational and the matter Lagrangians, respectively. As we are interested only in the gravitational tensor and the scalar field perturbations in this paper, we only display those terms here. In the absence of sources, the other modes can be gauged away. For further details and the intricacies of all perturbations we refer the reader to [20].

The tensor perturbations t_n have the Euclidean action

$$S_n^E = \frac{1}{2} \int t_n \hat{D} t_n + \ boundary \ term \tag{2.25}$$

where

$$\hat{D} = \left(-\frac{d}{d\tau} \left[a^3 \frac{d}{d\tau} \right] + a(n^2 - 1) \right) \tag{2.26}$$

where the background satisfied the classical equation of motion. The action is extremized when t_n satisfies the equation $\hat{D} t_n = 0$. Setting $N_0 = 1$, this is just the equation

$$\frac{d}{dt} \left[a^3 \dot{t}_n + (n^2 - 1) a t_n \right] = 0 \tag{2.27}$$

For t_n that satisfies the equation of motion, the action is just the boundary term

$$S_n^{E(cl)} = \frac{1}{2}a^3 \left(t_n \dot{t}_n + 4\frac{\dot{a}}{a}t_n^2 \right) \tag{2.28}$$

The path integral over t_n will be

$$\int d[t_n] \exp(-S_n^{E(cl)}) = (\det \hat{D})^{-1/2} \exp(-S_n^{E(cl)}) \tag{2.29}$$

Scalar fields can be treated in a similar fashion. The scalar field perturbation Lagrangian is given by

$$\frac{1}{2}N_0 a^3 \left[\frac{1}{N_0^2}\dot{f}_n^{\,2} - m^2 f_n^2 - \frac{(n^2-1)}{a^2}f_n^2 \right] \tag{2.30}$$

where we work in an appropriate gauge choice. Setting $N_0 = 1$, the equation of motion for f_n is

$$\frac{d}{dt}\left(a^3 \dot{f}_n \right) + \left[m^2 a^3 + (n^2-1)a \right] f_n = 0 \tag{2.31}$$

One can evaluate the scalar field path integral and the results are similar to that of the tensor perturbative modes. So in this paper we deal only with the gravitational tensor modes.

3. Perturbative Correction to the Bounce : No Back Reaction

Before doing the calculation that includes the backreaction due to the perturbative modes, we first perform a simple calculation to illustrate the key issues we are facing. The result of this section is meant to be a warm up and indicates the possibility of a correction to the wavefunction. We consider a S^4 solution with both its polar regions flattened by a small parameter δ and show that this squashed geometry allows for extra perturbative modes that can have significant effect on the calculation of the wavefunction. When $\delta = 0$, these extra perturbative modes vanish and what remains is the well known one-loop correction to the S^4. For a proper treatment the reader may go directly to the next section and onward.

The Euclidean equation of motion for the nth tensor perturbation mode on a S^4 background follows from Eq.(2.27)

$$\ddot{t}_n + 3\frac{\dot{a}_o}{a_o}\dot{t}_n - \left(\frac{(n^2-1)}{a_o^2} \right) t_n = 0 \tag{3.1}$$

where dot denotes differentiation with respect to the τ variable and $a_o(\tau) = \lambda^{-1/2}\sin(\sqrt{\lambda}\tau)$. This can be converted to a more familiar form by the substitution $F_n = \sqrt{\lambda}a_o(\tau)t_n$. In terms of F_n and $x = \cos(\lambda\tau)$ the equation of motion becomes

$$(1-x^2)\frac{d^2F_n}{dx^2} - 2x\frac{dF_n}{dx} + \left(2 - \frac{n^2}{(1-x^2)} \right) F_n = 0 \tag{3.2}$$

which is just an associate Legendre equation of degree one and order n. This has two linearly independent solutions, $P_1^{-n}(x)$ and $Q_1^n(x)$. However, because $a_o = 0$ at $x = \pm 1$, we have

$$S_E^n = \frac{1}{2} \left[a(\tau)^3 t_n \frac{dt_n}{d\tau} \right]_{x=1}^{x=-1} = 0 \tag{3.3}$$

We shall see in the following sections that once the perturbative modes are properly accounted for, S^4 changes to a "barrel". Anticipating this result we consider the S^4 instanton that is flattened slightly at the two poles. This "squashed" S^4 is given by $a_o(\tau) = \frac{1}{\sqrt{\lambda}} \sin(\lambda\tau)$, with the restricted range $1 - \delta \geq x \geq -1 + \delta$. There is a good reason for considering the squashed geometry. The perturbative modes can be strong enough to change the geometry of S^4 (and as we shall see, they will). So fixing the geometry to S^4 before doing the perturbative analysis is too restrictive. The perturbative modes on S^4 have to vanish at the two poles if they are to respect the background geometry. Their effect has been studied in [11,32] and apart from contributing to one-loop effect they do not lead to any tunneling suppression. However, to get the decoherence effect one must include modes that have nonvanishing values at the two poles. And we shall see that the squashed geometry allows for modes that can potentially lead to decoherence. This expectation will be confirmed in the later sections.

As explained in the introduction (Eq.(1.6,1.7)), to trace out a given perturbative mode (say, t_n) one must perform a path integral over that mode with the initial and final amplitudes the same (say, $t_n^i = t_n^f$), and then integrate over all possible values of t_n^i. This is just the trace operation in the path integral formalism. To do this we first find the action for t_n (details can be found in the appendix)

$$\begin{aligned} S_E^n &= \frac{1}{2} \left[a(\tau)^3 t_n \frac{dt_n}{d\tau} \right]_{x=(1-\delta)}^{x=(-1+\delta)} \\ &= \delta(2-\delta) \left[(1-\delta) + \delta(2-\delta) \frac{\partial_x Q_1^n(1-\delta)}{Q_1^n(1-\delta)} \right] (t_n^i)^2 \end{aligned} \tag{3.4}$$

Next we must find the prefactor to the path integral for t_n. This can be found as explained in the Appendix B. The prefactor is given by

$$\frac{1}{\sqrt{4\pi}} \frac{\sqrt{Q_1^n(1-\delta)P_1^n(1-\delta)(2\delta - \delta^2)}}{2^n} \sqrt{\frac{\Gamma\left(\frac{2-n}{2}\right)\Gamma\left(\frac{3-n}{2}\right)}{\Gamma\left(\frac{2+n}{2}\right)\Gamma\left(\frac{3+n}{2}\right)}} \tag{3.5}$$

Integrating over all initial states t_n^i (tracing out the nth mode) gives

$$\int dt_n^i K\left(t_n^i, x = -1 + \delta; t_n^i, x = 1 - \delta\right) = \alpha(n)f(\delta)2^{-n} \tag{3.6}$$

where $\alpha(n)$ contains factors of n and $f(\delta)$ is a function of δ, with $f(0) = 0$. $K\left(t_n^i, -1 + \delta; t_n^i, 1 - \delta\right)$ is the path integral over t_n mode with the boundary conditions $t_n(x = -1 + \delta) = t_n(x =$

$1 - \delta) = t_n^i$. Taking care of all the modes by tracing over all of them (within the lower and upper cut-offs), we get

$$\prod_n \int dt_n^i K\left(t_n^i, -1 + \delta; t_n^i, 1 - \delta\right) = \prod_n \alpha(n) f(\delta) 2^{-n} \tag{3.7}$$

$$= A(n, \delta) e^{-N^4/2 \ln(2)}$$

where N counts the modes. As explained later,

$$N = \left(\frac{H^{-1}}{l_s}\right) = 2^{1/4} \frac{\nu^{1/4}}{\sqrt{\lambda}} \tag{3.8}$$

where $\nu = \frac{2}{9\pi^2} \frac{G^2}{l_s^4}$. The wavefunction then corresponds to the path integral over the scale factor a and the perturbative modes t_n with the full action as given in Eq.(2.22). We trace over the perturbative modes and get

$$\Psi \simeq e^{\left(\frac{1}{3\lambda} - D\right)} \tag{3.9}$$

where D is the decoherence term leading to tunneling suppression

$$D \simeq \frac{\nu}{\lambda^2} \tag{3.10}$$

so the above result leads to an order $O(1/\lambda^2)$ suppression to the Hartle-Hawking wavefunction. This naive (naive because, as it turns out, the backreaction is important and also leads to an $O(1/\lambda^2)$ contribution) expectation is indeed vindicated in our calculation of the modified bounce. The more careful calculation changes the coefficient $\ln(2)$, but maintains the inverse square dependence on λ.

4. The Modified Bounce : Including Backreaction

The bounce solution S^4 is a solution to the Euclidean Einstein equation which is obtained as the Euler-Lagrange equation from the Euclidean action $S_0^E[a]$.

$$-2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = 3\lambda \tag{4.1}$$

The solution is given by $a(\tau) = \sqrt{\frac{1}{\lambda}} \cos(\sqrt{\lambda}\tau)$. This is the bounce in the absence of any perturbation. Including the perturbations, treating them as the environment, and tracing them out (as explained in Eq.(1.6,1.7)) will lead to a modified equation of motion instead of Eq.(4.1). In this section we derive this modified bounce equation.

Let us consider the effect of the metric perturbations. To find the modified bounce equation we have to carry out the path integral over the perturbation modes t_n is Eq. (2.29). Let us write down the path integral for a single tensor perturbation mode

$$\int D[t_n(\tau)] \exp\left(-\frac{1}{2} \int_{-T/2}^{T/2} d\tau\, a^3 [\dot{t_n}^2 + \frac{(n^2 - 1)}{a^2} t_n^2]\right) \tag{4.2}$$

This is a path integral for an oscillator with a varying mass as well as frequency. We can simplify this to a path integral of an oscillator with constant mass and variable frequency (given by $\omega_n(a(u)) = \sqrt{(n^2-1)}a(u)^2)$ using a new variable u

$$du = \frac{d\tau}{a(\tau)^3} \tag{4.3}$$

The Euclidean action now becomes

$$S_E = \frac{1}{2} \int_{u_i}^{u_f} du \left(t_n'^2 + (n^2-1)a^4 t_n^2\right) \tag{4.4}$$

$$= \frac{1}{2} \int_{u_i}^{u_f} du \left(-tt'' + \omega_n^2 t^2\right) + \frac{1}{2}(tt')|_{u_i}^{u_f}$$

To keep notation uncluttered, we drop the subscript n for the time being. Let $t = t_{cl} + \hat{t}$, where t_{cl} is a solution to the classical equation of motion for the above action

$$t_{cl}'' - \omega_n(a)^2 t_{cl} = 0 \tag{4.5}$$

with $t_{cl}(u_i) = t(u_i)$ and $t_{cl}(u_f) = t(u_f)$. Here $\hat{t}$ denotes fluctuations about the classical solution with $\hat{t}(u_i) = \hat{t}(u_f) = 0$. That is, $(tt')|_{u_i}^{u_f} = (t_{cl}\hat{t}')|_{u_i}^{u_f} + (t_{cl}t_{cl}')|_{u_i}^{u_f}$. Apriori, the contributions to the path integral will come from t_{cl} and $\hat{t}$. Here, the fluctuations $\hat{t}$ will lead to a prefactor. We shall keep track of this prefactor as it will have important contribution to the modified action. Substituting $t = t_{cl} + \hat{t}$ in Eq.(4.4), we obtain

$$S_E = \frac{1}{2} \int_{u_i}^{u_f} du \left(-\hat{t}\hat{t}'' + \omega_n^2 \tilde{t}^2\right) + (t_{cl}t_{cl}')|_{u_i}^{u_f} \tag{4.6}$$

The second term in the above equation is simply $S_E(t_{cl})$. The path integral in Eq.(4.2) is, therefore, given by

$$e^{-S_E(t_{cl})} P[\hat{t}] = e^{-S_E(t_{cl})} \int_0^0 D[\hat{t}(u)] \exp\left(-\frac{1}{2} \int_{u_i}^{u_f} du \left(-\hat{t}\hat{t}'' + \omega_n^2 \hat{t}^2\right)\right) \tag{4.7}$$

Notice that the prefactor $P[\hat{t}]$ is independent of $t(u_i)$ or $t(u_f)$. So the reduced path integral Eq.(1.7) becomes

$$Tr[Z] = \int D[a] \prod_n P[\hat{t_n}] \int dt_n^i \int dt_n^f \delta(t_n^i - t_n^f) e^{-S_E(t_{n,cl})} \tag{4.8}$$

We shall first evaluate the integral over $t_n^i = t_n^f$. In Appenxix C we evaluate the prefactor $P[\hat{t_n}]$. It is straightforward to include a scalar field.

Although finding a general solution to Eq.(4.5) is in general impossible, we note that it has the same form as the Schrodinger equation for a particle in a potential $\omega_n(a(u))$. So for a slowly varying potential we can use the WKB method for finding the solution to Eq.(4.5). By slowly varying one means that the following condition is satisfied

$$\frac{d\omega_n}{d\tau} << \omega_n^2 \tag{4.9}$$

This is satisfied by the higher modes $n >> 1$ and we shall see that these modes are the ones that contribute to the suppression of quantum tunneling. (One may be concerned that $\omega_n \to 0$ as $a \to 0$. As we shall see, in contrast to the unperturbed bounce, a actually stays finite in the modified bounce. In this sense, the WKB approximation is reasonable.) The two independent WKB solutions to Eq.(4.5) are

$$t_{cl}^{\pm} = \frac{1}{\sqrt{\omega_n}} \exp\left(\pm \int^u du' \omega_n(u')\right) \tag{4.10}$$

The general solution will be a linear combination of the independent solutions

$$t_{cl}(u) = A t_{cl}^+(u) + B t_{cl}^-(u) \tag{4.11}$$

The values of A and B will depend on the boundary conditions. If Eq.(4.11) is to satisfy the following boundary conditions

$$t_{cl}(u_i) = t^i \ t_{cl}(u_f) = t^f \tag{4.12}$$

then we have the following values of A and B

$$A = \frac{t^f \sqrt{\omega_n(u_f)} - t^i \sqrt{\omega_n(u_i)} \exp(-D_n)}{(\exp(D_n) - \exp(-D_n))} \tag{4.13}$$

$$B = -\frac{t^f \sqrt{\omega_n(u_f)} - t^i \sqrt{\omega_n(u_i)} \exp(D_n)}{(\exp(D_n) - \exp(-D_n))}$$

where D_n is given by

$$D_n = \int_{u_i}^{u_f} du \ \omega_n(a(u)) = \int_{-T/2}^{T/2} d\tau \ \frac{\omega_n(a(\tau))}{a(\tau)^3} \tag{4.14}$$

Next, substituting the solution Eq.(4.11), using Eq.(B.2), in Eq.(4.6), we get

$$S_E(t_{cl}) = \tfrac{1}{2}\left(t_{cl}(u_f)\frac{dt_{cl}(u_f)}{du} - t_{cl}(u_i)\frac{dt_{cl}(u_i)}{du}\right) \tag{4.15}$$

$$= \frac{1}{2(\exp(D_n) - \exp(-D_n))}\left[\left((t^f)^2\omega_n(u_f) + (t^i)^2\omega_n(u_i)\right)(\exp(D_n) + \exp(-D_n))\right.$$
$$\left. - 4t^i t^f \sqrt{\omega_n(u_i)\omega_n(u_f)}\right]$$

This gives the classical contribution Eq.(4.7). Note that in order to perform the trace over this perturbation mode, we will be setting $t^i = t^f$, and $\omega_n(u_i) = \omega_n(u_f)$. The path integral over $\hat{t}$ leads to the prefactor. Its evaluation for a time-dependent frequency is explained in Appendix C (Eq.(C.24)). (Equivalently, one may use the Pauli-van Vleck-Morette formula studied by Barvinsky [26]). The prefactor (path integral over $\hat{t}$) gives

$$\int D[\hat{t}(u)] \exp\left(\frac{1}{2}\int_{u_i}^{u_f} du\left(-\hat{t}\hat{t}'' + \omega_n^2\hat{t}^2\right)\right) = \frac{\sqrt{\omega_n(u_f)}}{\sqrt{(\exp(D_n) - \exp(-D_n))}} \tag{4.16}$$

Now the trace over the perturbative modes can be performed by setting $t^i = t^f$ and integrating over the amplitudes t^i

$$\int dt^f \int dt^i \delta(t^i - t^f) \int_{t^i}^{t^f} D[t(u)] \exp\left(-S_E\right) \tag{4.17}$$

$$= \frac{\sqrt{\omega_n(u_f)}}{\sqrt{(\exp(D_n) - \exp(-D_n))}} \int dt^f \exp\left(-S_E[t_{cl}]\right)$$

$$\simeq \frac{1}{\sqrt{(\exp(D_n) - \exp(-D_n))}}$$

where we used $\int dt^f \exp(-S_E[t_{cl}]) \simeq 1/\sqrt{\omega_n(u_f)}$. Also, note that $D_n >> 1$ for large values of n, and $(\exp(D_n) - \exp(-D_n))^{-1/2} \simeq \exp(-D_n/2)$. Finally, one has to perform this tracing-over over all n modes. This leads to the following contribution

$$\prod_n \frac{1}{\sqrt{(\exp(D_n) - \exp(-D_n))}} \simeq \exp(-\sum \frac{D_n}{2}) \tag{4.18}$$

From the expansion in Eq.(2.18) it is clear that one has to count the indices n, l, m (corresponding to the spherical harmonics on S^3). For a given n, l can take $n-2$ values ($l = 2, 3, ..., n-1$), and for a given l, m can take $2l+1$ values ($m = -l, -l+1, ..., l-1, l$). For a given n this introduces a degeneracy of $f(n) \simeq 2n^2$. The factor of 2 comes due to the two tensor modes t^0_{nlm} and t^e_{nlm} in the expansion in Eq.(2.18). To proceed further, we need to define the cut-offs. A natural short wavelength cut-off is the string scale, l_s. The long wavelength cut-off is the inverse Hubble length. This gives a cut-off for n

$$n_{max} = N = \left(\frac{H^{-1}}{l_s}\right) = \frac{1}{l_s}\sqrt{\frac{3}{\Lambda}} \tag{4.19}$$

$$D \equiv \sum \frac{D_n}{2} = \frac{1}{2}\sum_{n,l,m}^{N} \int_{-T/2}^{T/2} d\tau \frac{\omega_n(a)}{a^3} \tag{4.20}$$

where we have used Eq.(4.14). Note that $\omega_n(a)^2 = (n^2-1)a^4$. For higher modes, therefore, $\omega_n \simeq na^2$. Thus,

$$D = \frac{1}{2}\int_{-T/2}^{T/2}\frac{d\tau}{a}\sum_{n,l,m}^{N} n \tag{4.21}$$

$$= \frac{N^4}{8}\int_{-T/2}^{T/2}\frac{d\tau}{a} = \frac{1}{2}\int_{-T/2}^{T/2}d\tau\frac{\nu}{\lambda^2 a}$$

We can also easily generalize to arbitrary number of degrees of freedom, with

$$\frac{\nu}{\lambda^2} = \frac{n_{dof}N^4}{4} = \frac{n_{dof}}{4(Hl_s)^4} \tag{4.22}$$

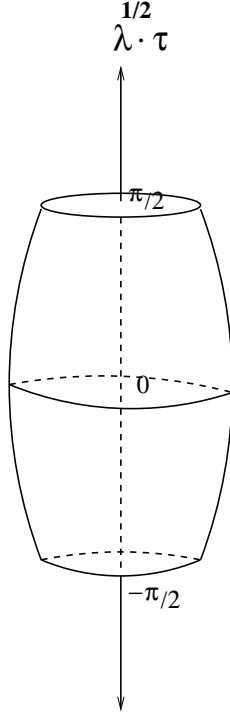For pure gravity, we have $n_{dof} = 2$. The modifed Euclidean action is, therefore, given by

$$S_{E,dC} = S_{E,0} + D[a] = \frac{1}{2}\int d\tau\left(-a\dot{a}^2 - a + \lambda a^3 + \frac{\nu}{\lambda^2 a}\right) \tag{4.23}$$

To evaluate $S_{E,dC}$ via the saddle-point approximation, we have to find the classical path, or the bounce solution, of this effective action.

5. The Bounce Solution

The equation of motion for the Euclidean action, Eq.(4.23), is

$$-2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = 3\lambda - \frac{\nu}{\lambda^2 a^4} \tag{5.1}$$

Figure 6: The Modified Bounce : It is a squashed version of S^4 resulting in a "barrel".

This is just the Euclidean version of the Einstein equation with both a cosmological constant and radiation. As is well known, this equation allows for a variety of solutions (M1, M2, A1, A2, E, O1) [27]. One has to be careful in deciding which one of these is the correct bounce solution. The bounce solution is the real solution to the classical Euclidean equation of motion. The M2 solution satisfies the bounce criteria and is given by (in the Euclidean form)

$$a(\tau) = \frac{1}{\sqrt{2\lambda}}\sqrt{\left(1 + \sqrt{\left(1 - \frac{4\nu}{\lambda}\right)\cos(2\sqrt{\lambda}\tau)}\right)} \tag{5.2}$$

This solution is the modified bounce and has some interesting features.

- It is a deformation of the S^4 instanton from spherical to barrel-shaped, with $-\pi/2\sqrt{\lambda} \leq \tau \leq \pi/2\sqrt{\lambda}$. It reduces to the usual S^4 instanton when $\nu = 0$, as expected

$$a(\tau) \rightarrow \frac{1}{\sqrt{\lambda}}\cos(\sqrt{\lambda}\tau) \tag{5.3}$$

  Here ν is the deformation parameter.

- For large values of ν the bounce is destroyed. Presumably, there is no more quantum tunneling because of excessive decoherence. This critical value of λ is given by $\lambda_c = 4\nu$. For tunneling $\lambda > \lambda_c$.
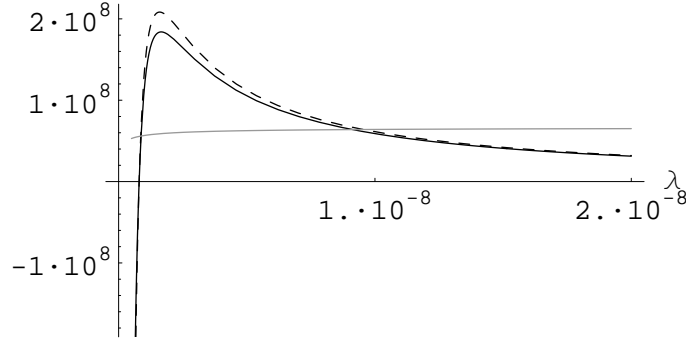
Figure 7: The solid curve is $F = -S_{E,dC}$, using the exact value of $\Im$. The dotted curve shows F using $\Im \simeq 8/3$. The light flat curve is the integral $\Im$, which asymptotes to 8/3 for large λ. The plots are given with $M_{Pl}/M_s = 10^3$. The dotted and the solid curves differ only slightly, justifying the approximation made.

The modified Euclidean action can now be calculated. One has to be careful though to include the $O(\nu/\lambda^2)$ contribution from the $-a\dot{a}^2 - a + \lambda a^3$ part. This contribution will add to the $O(\nu/\lambda^2)$ contribution from the $\frac{\nu}{\lambda^2 a(\tau)}$ term and give the total $O(\nu/\lambda^2)$ modification. We state the final result

$$-F = S_{E,dC} = \left(-\frac{1}{4\lambda} + \frac{\nu}{\lambda^2}\right)\Im \tag{5.4}$$

where $\Im$ is an integral given by

$$\Im = \int_{\frac{-\pi}{2\sqrt{\lambda}}}^{\frac{\pi}{2\sqrt{\lambda}}} d\tau \left(\frac{\sin^2(2\sqrt{\lambda}\tau)}{a(\tau)}\right) \tag{5.5}$$

where $a(\tau)$ is the bounce solution given by Eq.(5.2). The exact value of $\Im$ involves an elliptic integral, but we can make a fairly accurate estimate and the result is

$$\Im \simeq \frac{8}{3} \tag{5.6}$$

For tunneling from nothing, we should consider the barrel to be a deformed S^4 with the same topology. That is, we should include the contribution of end plates of the barrel in the evaluation of $S_{E,dC}$ in Eq.(5.4). At the end plates,

$$a = \frac{1}{\sqrt{2\lambda}}\left(1 - \sqrt{1 - \frac{4\nu}{\lambda}}\right)^{1/2} > 0$$

However, their contribution to $S_{E,dC}$ is zero because the weighing factor $\sin^2(2\sqrt{\lambda}\tau)$ in Eq.(5.5) vanishes at the end plates.

As expected, for $\nu = 0$ we get the usual Hartle-Hawking wavefunction. Now, including the environment, the tunneling probability is given by

$$P \simeq e^{\left(\frac{2}{3\lambda} - \frac{8\nu}{3\lambda^2}\right)} \tag{5.7}$$

For $\lambda \to 0$, higher order decoherence effects should be included. These effects can come from a more careful treatment of the leading term we have obtained, or from higher order interaction of the modes with a and among themselves. Quantum effects may become relevant. This is especially so at large λ. Note that $S_{E,dC}$ peaks around $\lambda = 8\nu$. So for regions with non-zero tunneling probability, $\lambda > \lambda_c = 4\nu$.

6. The Modified Hamilton Constraint and the Wheeler-DeWitt Equation

The modified action, as given in Eq.(4.23), leads to a modified Hamiltonian constraint. The modified Lorentzian action is given by

$$S = \frac{1}{2} \int d\tau \left(-a\dot{a}^2 + a - \lambda a^3 - \frac{\nu}{\lambda^2 a}\right) \tag{6.1}$$

The modified Hamiltonian constraint is

$$H = \frac{1}{2a}\left(-\Pi_a^2 - a^2 + \lambda a^4 + \frac{\nu}{\lambda^2}\right) = 0 \tag{6.2}$$

where $\Pi_a = -a\dot{a}$ is the conjugate momentum. Using the Hamiltonian constraint and the equation of motion following from the modified Lorentzian action, we have

$$(\frac{\dot{a}}{a})^2 + \frac{1}{a^2} = \lambda + \frac{\nu}{\lambda^2 a^4} \tag{6.3}$$
$$\frac{\ddot{a}}{a} = \lambda - \frac{\nu}{\lambda^2 a^4}$$

Since $\ddot{a}/a = -\sum(\rho_i + 3p_i)/2 = -\sum \rho_i(1 + 3\omega_i)/2$, this implies that the new ν term has equation of state $\omega = 1/3$, precisely that of ordinary radiation.

One can write down a differential equation describing the evolution of the wavefunction - the Wheeler-DeWitt (WdW) equation -by imposing the condition as an operator equation. This is quantum gravitational analog of the Schrodinger equation in quantum mechanics.
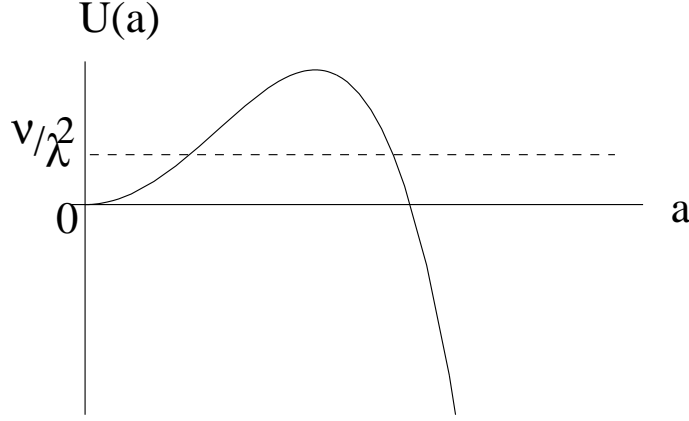
$$-\hat{H}\Psi = \left(\frac{\hat{\Pi}_a^2}{2} + U(a)\right)\Psi = 0$$

where $\hat{\Pi}_a = -i\partial/\partial a$ and $U(a)$ is the gravitational potential given by

$$U(a) = U_0(a) - \frac{\nu}{\lambda^2} = a^2 - \lambda a^4 - \frac{\nu}{\lambda^2} \tag{6.4}$$

Recall the case without the ν term. In the classically forbidden region, i.e., the under-barrier region $\lambda^{-1/2} \geq a \geq 0$, the WKB solutions for the tunneling amplitude from $a = 0$ to $a = \lambda^{-1/2}$ are

$$\Psi_{\pm} \simeq e^{\pm \int_0^{1/\sqrt{\lambda}} |\hat{\Pi}(a')|da'} \tag{6.5}$$

Figure 8: The solid curve is $U_0(a) = a^2 - \lambda a^4$. The effect of the new term is simply to raise the energy eigenvalue from 0 to $E = \nu/\lambda^2 > 0$.

Note the Hartle-Hawking no boundary prescription requires us to take the positive sign in the exponent in Eq.(6.5), that is, Ψ_+. This yields the HH wavefunction. Including the ν term is like solving the wave equation with a positive (instead of zero) energy eigenvalue $E = \nu/\lambda^2$ (see Figure (8)),

$$\left(\frac{\hat{\Pi}_a^2}{2} + U_0(a) \right) \Psi(a) = E\Psi(a)$$

The presence of $E > 0$ decreases the tunneling amplitude Ψ_+. To see the origin of E, consider the presence of a generic field χ, so the WdW equation is crudely given by

$$\left(-\frac{\hat{\Pi}_a^2}{2} - U_0(a) + \frac{\Pi_\chi^2}{2} + \omega(\chi, a)^2 \right) \Psi(a, \chi) = 0 \tag{6.6}$$

Note that the kinetic terms in Eq.(6.6) have opposite signs. Let

$$\Psi(a, \chi) = \Sigma_n \psi_n(a) u_n(\chi) \tag{6.7}$$

then the above equation can be separated,

$$\left(\frac{\Pi_\chi^2}{2} + \omega(\chi, a)^2 \right) u_n(\chi) = \epsilon_n(a) u_n(\chi) \tag{6.8}$$

$$\left(-\frac{1}{2} \frac{d^2 \psi_n}{da^2} + U_0(a)\psi_n \right) = \epsilon_n(a)\psi_n$$

Taking the ground state, we obtain the zero point energy ϵ_0, with $\Psi(a) = \psi_0(a)$. Including the zero point energy of all χ fields then yields E, which turns out to be independent of a.

If instead, we take Ψ_- in Eq.(6.5), as suggested by Linde and Vilenkin [8] and which is more familiar in quantum mechanics, we see that the inclusion of the ν term enhances tunneling. This means interaction with the environment enhances the tunneling amplitude,

which is counter-intuitive. Furthermore, the highest tunneling probability will go for large Λ, when the semi-classical approximation used here breaks down. We believe the Hartle-Hawking no boundary prescription is the correct one for the spontaneous creation of the universe.

As we see clearly now, this ν term plays the role of radiation. The deformed S^4 has, in a sense, allow the spontaneous creation of a universe with some radiation in it. In pure gravity, this is simply gravitational radiation. Due to the presence of such radiation, the cosmic scale factor $a(\tau)$ does not vanish any more at the poles of S^4. In fact, the decoherence has led to a flattening of the two poles.

We note that, in theories with dynamical λ, there will be momentum terms associated with λ in the Wheeler-DeWitt equation. For example, if λ is associated with the potential energy of some scalar field ϕ, then there will be momentum term $\hat{\Pi}_\phi$ in the Wheeler-DeWitt equation. However, the inclusion of such a momentum term for λ is not expected to change the boundedness of the gravitational potential.

For large cosmological constant Λ, the radiation is suppressed. In this case, one may ignore it. However, as Λ decreases and the size of the universe grows, radiation becomes important. More radiation also means stronger suppression of the tunneling, consistent with the intuitive picture of tunneling while interacting with the environment. It is this suppression that provides a bound to the Euclidean gravity action.

7. Connection to SOUP

In [3] we lay out the motivation for SOUP, short for "Selection of the Original Universe Principle". This is an alternate to the Anthropic Principle. Since observational evidence of an inflationary epoch is very strong, we suggest that the selection of our particular vacuum state follows from the evolution of the inflationary epoch. That is, our particular vacuum site in the cosmic landscape must be at the end of a road that an inflationary universe will naturally follow. Any vacuum state that cannot be reached by (or connected to) an inflationary stage can be ignored in the search of candidate vacua. That is, the issue of the selection of our vacuum state becomes the question of the selection of an inflationary universe, or the selection of an original universe that eventually evolves to an inflationary universe, which then evolves to our universe today. The landscape of inflationary states/universes should be much better under control, since the inflationary scale is rather close to the string scale. In [3] we proposed that, by analyzing all known string vacua and string inflationary scenarios, one may be able to pin down SOUP. Here, the rate of tunneling from nothing (i.e., no classical spacetime) to a deSitter universe (much like the inflationary universe) is now given by $\Gamma \simeq \exp(F) = \exp(-S_{E,dC})$, where

$$-F = S_{E,dC} = S_{E,0} + D \simeq -\frac{3\pi}{G\Lambda} + \frac{n_{dof}}{4\pi^2}\frac{V_4}{l_s^4} \tag{7.1}$$

where D is the decoherence term, l_s is the cut-off scale and n_{dof} is the number of light degrees of freedom. Here V_4 is the 4-volume of the instanton. For S^4, $V_4 = 8\pi^2 r^4/3 = 24\pi^2/\Lambda^2$ is its area. In string theory, that cut-off is naturally provided by the string scale. Note that $S_{E,dC}$ is now bounded from below.

To gain some idea of the magnitude of F, we use Eq.(1.9) to crudely estimate the value of Λ with maximum tunneling probability,

$$\Lambda_{max} \simeq \frac{4n_{dof}G}{\pi l_s^4}$$

so that

$$F_{max} \simeq \frac{3}{2n_{dof}}(2\pi M_{Pl}l_s)^4$$

For the string scale a few orders of magnitude smaller than M_{Pl}, we see that Λ_{max} takes a value quite close to that expected in an inflationary universe, with $F \sim 10^{12}$. On the other hand, the critical value of Λ is $\Lambda_c \simeq \Lambda_{max}/2$. For $\Lambda < \Lambda_c$, the barrel-shaped instanton is destroyed. At Λ_c, $F(\Lambda_c) \simeq 0$, so the tunneling probability at Λ close to Λ_c is already negligibly small compared to that at Λ_{max}. Tunneling to a supersymmetric vacuum is totally suppressed. For l_s around the TeV scale [28], we find that Λ_{max} is quite close to today's dark energy value. This is similar in spirit to Ref [29]. However, this scenario will imply that our universe has not really gone through the whole hot big bang and certainly not the standard inflationary epoch.

The key tool is the modified bounce. Although we only deal with the S^4 instanton and its modification in this paper, one can easily generalize this analysis to higher dimensions. We expect the form of the tunneling probability to be $P \sim e^F$, with $F = -S_{E,dC} = -S_E - D$. In 10-D,

$$S_{E,dC} \simeq S_{E,10} + c\left(\frac{V_{10}}{l_s^{10}}\right) \tag{7.2}$$

where $S_{E,10}$ is the 10-D Euclidean action determined in mini-superspace and V_{10} is the 10-dimensional volume of the instanton. In effective 4-D theory, $S_{E,10}$ reduces to $-3\pi/G\Lambda$, since

$$8\pi G = M_{Pl}^{-2} = \frac{g_s^2 l_s^8}{4\pi V_6}$$

where V_6 is the 6-D compactification volume. (Recall $\alpha' = M_s^{-2} = (l_s/2\pi)^2$.) The constant c has to be calculated for each vacuum and will depend on the details of the vacuum. To get an order of magnitude estimate of c we have, by comparing to the 4-D case,

$$c \simeq \frac{n_{dof}}{\pi}\frac{1}{M_{Pl}^2 l_s^2 g_s^2} \tag{7.3}$$

so we expect c to be small. In [3] we have considered such instantons. We did not do a careful calculation of ν then, that has been the main content of the present paper, but we phenomenologically guesstimated a possible range for its values and calculated the creation probabilities for various vacua in the cosmic landscape. Among all the known string vacua, we find that the universe most likely to be created is a KKLMMT type inflationary vacuum. (For details, we refer the reader to [3].) The probabilities calculated were very robust and did not depend on the exact value of c. Changing from V_9 to V_{10} does not change the overall qualitative results. Therefore, we expect the conclusions to stay the same.

There are some minor differences. There we find that tunneling to a universe with today's dark energy is very much suppressed. Here we see that the cosmological constant corresponding to today's dark energy is actually below the critical value, so tunneling directly to today's universe is simply zero. A more careful calculation based on Eq.(7.2) is clearly needed. However, the important point is that one can calculate it, at least in principle, for any given vacuum and, therefore, compare their probabilities. This would be a huge improvement over the Anthropic Principle.

One can calculate the tunnelling probability to S^{10} using Eq.(7.2). For S^{10}, $S_{E,10} = \frac{\Lambda V_{10}}{32\pi G_{10}}$, where $V_{10} = \frac{2^{11} 5! \pi^5}{10!}/H^{10}$, is the volume of S^{10}. Note that for S^{10}, $H^2 = \Lambda/36$. Without branes, the solution we consider is a 10D supersymmetric vacuum. Tunneling to this vacuum is suppressed. Next we consider tunneling to a deSitter-like vacuum. Let there be N $D9$ brane-antibrane pairs in the S^{10}. So

$$\Lambda = 2N \times 8\pi G_{10} \times \frac{M_s^{10}}{(2\pi)^9 g_s} = \frac{g_s M_s^2 N}{(2\pi)^4} \tag{7.4}$$

Maximizing F in terms of N gives

$$N = \frac{5 n_{dof} (2\pi)^6}{g_s} \left(\frac{M_s}{M_{Pl}} \right)^2 \tag{7.5}$$

This means $N = 1$ is a reasonable choice for $g_s \sim 1$, and $M_s/M_{Pl} = 10^{-3}$. One can then calculate F for $N = 1$ for S^{10}. In this case, F is dominated by $S_{E,0}$. One gets

$$F(S^{10}) \simeq -S_{E,0}(S^{10}) = \frac{2^{11} \, 5! \, (36\pi)^5}{10!} \simeq 4 \times 10^9 \tag{7.6}$$

Following Ref [3], we see that other related geometries such as $S^5 \times S^5$, $S^4 \times S^6$, $S^4 \times S^3 \times S^3$ etc. have slightly smaller but similar values of F.

For a KKLMMT-like inflationary universe, the choice of fluxes fixes the string scale as well as the scale of inflation, where G and the density perturbation measured in the cosmic microwave background radiation are used as input parameters. Using Eq.(7.1) and $\Lambda = 8\pi G \rho_{vac}$, where the compactified volume has been absorbed into G, one can rewrite Eq.(7.1) so

$$F = \frac{3}{8} \left(\frac{\sqrt{8\pi} M_{Pl}}{\rho_{vac}^{1/4}} \right)^4 - \frac{3}{4(2\pi)^6} \left(\frac{M_s}{\sqrt{8\pi} M_{Pl}} \right)^4 \left(\frac{\sqrt{8\pi} M_{Pl}}{\rho_{vac}^{1/4}} \right)^8 \tag{7.7}$$

Plugging in the above mentioned values for the mass scales, we get F. For a typical choice of fluxes that would give $M_s \sim 10^{15}$GeV and an inflationary scale $\rho_{vac}^{1/4} \sim 10^{14}$GeV, with the Planck mass $M_{Pl} \sim 10^{18}$GeV, one gets

$$F \sim 10^{16}$$

for a realistic brane inflationary scenario. This big increase in F from the 10D scenario is a result of the decrease in the effective cosmological constant due to the warping of the geometry. By varying the RR and the NS-NS fluxes, one can maximize F in a way which is not possible in 10D. Of course, this value is sensitive to the details of the model. A more careful calculation will be important.

8. Discussion

Let us make some comments here.

- The loop correction to the Euclidean action has been calculated before [10, 11]. It has the form $\log(\Lambda/\mu)$, so S_E becomes

$$S_E = -\frac{3\pi}{G\Lambda} \rightarrow -\frac{3\pi}{G\Lambda}\left(1 + \beta G\Lambda \log(\Lambda/\mu)\right)$$

where β measures the number of fields involved. Ref [32] considers the metric perturbation also; however, they then relate it to the one-loop contribution, yielding the above result. Note that this loop correction does not solve the boundedness problem in the HH wavefunction. Its implications on probability of inflation has also been discussed (see [33] and the references therein).

- Decoherence effects are to be distinguished from the particle creation effects that have been discussed in the literature [9]. Particle creation effects arise at the one-loop level, while the decoherence effect discussed here is a back-reaction improved quantum correction.

- Back reaction is crucial in obtaining the decoherence term. This reminds one of the situation in quantum field theory, where a simple 1-loop correction to the coupling takes on new significance when it is renormalization group improved.

- Integrating out the environment provides a new interaction term for the cosmic scale factor a in the Einstein equation. This decoherence term behaves just like ordinary radiation and generates an effective term in the Euclidean action for the instanton. The presence of this term provides the lower bound to the resulting Euclidean gravity.

- The correction to the Hartle-Hawking wavefunction results in a decrease in the Euclidean action. If we are to interpret the Euclidean action as the entropy, then there is a decrease in the entropy due to decoherence. This is in accordance with the Bekenstein bound on entropy. A pure deSitter space provides the upper bound on entropy. Decoherence, which is due to the inclusion of extra degrees of freedom, then leads to a lower entropy respecting the entropy bound.

- Once the inflationary universe is created, inflation simply red-shifts the radiation so the radiation term quickly becomes negligible. It seems that the initial radiation is present just to suppress the tunneling.

- It is interesting to speculate what happens after inflation. Towards the end of inflation, the inflaton field (or the associated tachyon mode) rolls down to the bottom of the inflaton potential. The universe is expected to be heated up to start the hot big bang epoch. It is not unreasonable to expect the wavefunction of the universe to be a linear superposition of many vacua (say, a billion of them) at the foothill of the inflaton potential. This is like a Bloch wavefunction as proposed in Ref [35]. Since

tunneling happens only between vacua with (semi-)positive cosmological constants, this wavefunction should be a superposition of vacua with only positive cosmological constants. The ground state energy of this wavefunction is expected to be much smaller than the average vacuum energy, a property of Bloch wavefunctions. As the universe evolves, tunneling among the vacua will become suppressed, due to cooling as well as decoherence. Eventually, the environment collapses the wavefunction to a single vacuum. Since the Bloch wavefunction is able to sample many vacua, it is natural to expect that it will collapse to the vacuum state with the smallest positive vacuum energy within teh sample. This may partly explain why the dark energy is so small.

- In terms of recent work on the statistics of the landscape [30, 31], one may view our work as providing a measure to the counting of vacua. Each vacuum is weighted with its probability of tunneling from nothing. Since the exponent of the tunneling probability can differ by many orders of magnitude, this is likely to be the dominant contribution to the measure. We argue that a 4-dimensional inflationary universe very much like ours has large probability; this implies that vacua that cannot be reached after inflation should have vanishing measure.

- The effect of decoherence on other tunneling problems in quantum gravity should be re-examined. These include tunneling in eternal inflation, the Coleman-deLuccia tunneling etc.

9. Summary and Remarks

We address a number of questions in this paper.

First we ask what happens to the well known S^4 bounce solution to the Euclidean Einstein equations when the perturbations to the metric are taken into account. How does the bounce change as a result? In particular, is there a parameter characterizing the perturbations that describes the modification of the bounce? Is there a range of this parameter that destroys the bounce altogether?

To answer the first question, we apply the path integral techniques to trace out the perturbations. The result is summarized in the introduction. Taking back-reaction into account, the Euclidean action now includes an additional term that encapsulates the effect of the perturbations. Depending on the value of the parameter c or equivalently ν, the bounce gets deformed. For a fairly large range of this parameter, the effect is to just deform the bounce solution. Since this contribution is always positive, the tunneling is always suppressed. At a critical value, the bounce is destroyed. One may interpret that tunneling is forbidden in this limit.

Next, we ask what these perturbations do to the boundedness of the Euclidean gravitational action. As is well known, the Euclidean gravitational action for a closed spacetime is not bounded from below for theories with a dynamical cosmological

constant. This manifests, for example, as the infinite peaking of the wavefunction of the universe at the vanishing value of Λ. Does the inclusion of the perturbations change anything here?

To answer this question, we see that the effect of the perturbations is to change the wavefunction of the universe from $\exp(3\pi/G\Lambda)$ to $\exp(3\pi/G\Lambda - C/\Lambda^2)$ where c depends on the particular perturbations that we are looking at. So, at least for the problem at hand, the inclusion of the perturbations and their back reaction makes the Euclidean action bounded from below. This renders the wavefunction normalizable. In the study of tunneling, it makes no sense to talk about gravity without taking into account the perturbations to the metric. In this sense, quantum gravity is consistent as long as we are careful to include the effects of the perturbative modes which are always present.

Once we have a sensible wavefunction, we can now go ahead and apply it to the cosmic landscape in string theory. Since Euclidean action and tunneling probability are both dimensionless, we can compare the tunneling from nothing to any point in the landscape and find the sites that have the largest probability. This program hopefully will allow us to understand why we are where we are, without resorting to the anthropic principle.

Acknowledgments

A. A Quantum Mechanical Example

To see the basic idea, recall the quantum tunneling of the system

$$L_0 = \frac{M}{2}\dot{q}^2 - V(q) \tag{A.1}$$

with a quartic $V(q)$ as shown in Fig. 2. The tunneling rate Γ from the local minimum at $q = 0$ to the exit point $q = q_0$ is well-known,

$$\Gamma = A\exp(-S_0)$$
$$S_0 = \int^{q_0} \sqrt{2MV(q)}dq = \int \left(\frac{M}{2}\dot{q}^2 + V(q)\right)d\tau \tag{A.2}$$

where τ is the Euclidean time and S_0 is the bounce, i.e., the instanton solution [13]. This WKB approximation is good provided that the height of the barrier is larger than ω_0, where

$$M\omega_0^2 = \frac{\partial^2 V}{\partial q^2}|_{q=0}$$

Note that, for $V(q)$ bounded from below, S_0 is bounded from below, as required by consistency. In a more realistic situation, the particle interacts with the environment. Typically, this introduces a frictional force, so the corresponding classical equation is given by

$$M\ddot{q} + \eta\dot{q} + \frac{\partial V}{\partial q} = 0 \tag{A.3}$$

The impact of such a frictional term on the particle is to suppress the tunneling rate. Consider the following system

$$L = \frac{M}{2}\dot{q}^2 - V(q) + \frac{1}{2}\sum_\alpha m_\alpha \left(\dot{x}_\alpha^2 - \omega_\alpha^2 x_\alpha^2\right) - q\sum_\alpha C_\alpha x_\alpha - \frac{1}{2}M(\delta\omega)^2 q^2 \tag{A.4}$$

where one may consider q to be the system and the x_α to be the environment. The last term is a counter term introduced to correct the shift in frequency,

$$M(\delta\omega)^2 = \sum_\alpha \frac{C_\alpha^2}{m_\alpha\omega_\alpha^2} \tag{A.5}$$

The interactions of q with the x_α introduces the friction term

$$\eta = \frac{\pi}{2}\sum_\alpha \frac{C_\alpha^2}{m_\alpha\omega_\alpha^2}\delta(\omega_\alpha - \omega) \tag{A.6}$$

for ω smaller than some critical ω_c.

The tunneling rate of this sytem can be easily found [14, 15] that the bounce S_0 increases to

$$S \simeq S_0 \left(1 + \frac{\eta}{2M\omega_0}\right) \tag{A.7}$$

That is, the interaction with the environment, or the friction, suppresses the tunneling rate. This qualitative feature remains true when the environment and its interaction with the system is more complicated.

B. Calculations on the Squashed S^4

This appendix gives details for Section 3. We show how we perform the path integral over the t_n mode. The solution to Eq.(3.1) satisfying the initial condition $t_n(x = 1 - \delta) = t_n^i$, and the final condition $t_n(x = -1 + \delta) = t_n^f$, is given by

$$t_n(x) = \frac{F_n(x)}{\sqrt{1 - x^2}} = \frac{AP_1^{-n}(x) + BQ_1^n(x)}{\sqrt{1 - x^2}} \tag{B.1}$$

where A and B are given by

$$A = \sqrt{2\delta - \delta^2} \frac{t_n^i Q_1^n(-1+\delta) - t_n^f Q_1^n(1-\delta)}{P_1^{-n}(1-\delta)Q_1^n(-1+\delta) - P_1^{-n}(-1+\delta)Q_1^n(1-\delta)} \tag{B.2}$$

$$B = -\sqrt{2\delta - \delta^2} \frac{t_n^i P_1^{-n}(-1+\delta) - t_n^f P_1^{-n}(1-\delta)}{P_1^{-n}(1-\delta)Q_1^n(-1+\delta) - P_1^{-n}(-1+\delta)Q_1^n(1-\delta)}$$

The trace operation is defined as doing the following

$$\int dt_n^i \int dt_n^f \delta(t_n^i - t_n^f) \int_{(t_n^{final}=t_n^f)}^{(t_n^{initial}=t_n^i)} D[t_n] \exp\left(S_E^n[t_n(x)]\right) \tag{B.3}$$

Since our purpose finally is to take to trace over the t_n mode, we can set $t_n^i = t_n^f$ at this stage. The Euclidean action due to the nth mode is then

$$S_E^n = \frac{1}{2}\left[a(\tau)^3 t_n \frac{dt_n}{d\tau}\right]_{x=(1-\delta)}^{x=(-1+\delta)} \tag{B.4}$$

$$= \frac{1}{2}\left[xF_n^2 + (1-x^2)F_n\frac{dF_n}{dx}\right]_{x=(1-\delta)}^{x=(-1+\delta)}$$

Using Eq.(B.1, B.2) and the symmetry properties of the associate Legendre functions

$$P_1^{-n}(-x) = -P_1^{-n}(x), \qquad Q_1^n(-x) = Q_1^n(x) \tag{B.5}$$

$$\partial_x P_1^{-n}(-x) = \partial_x P_1^{-n}(-x), \qquad \partial_x Q_1^n(-x) = -\partial_x Q_1^n(x)$$

one gets the following action

$$S_E^n = \delta(2-\delta)\left[(1-\delta) + \delta(2-\delta)\frac{\partial_x Q_1^n(1-\delta)}{Q_1^n(1-\delta)}\right](t_n^i)^2 \tag{B.6}$$

Next we must find the prefactor to the path integral for t_n. This can be found as explained in the Appendix C. The prefactor is given by $\frac{1}{\sqrt{2\pi g_n(-1+\delta)}}$, where $g_n(x)$ is a solution to Eq.(3.1), i.e. $g_n(x) = \frac{CP_1^{-n}(x)+DQ_1^n(x)}{\sqrt{1-x^2}}$, and it satisfies the following conditions

$$g_n(1-\delta) = 0, \quad \partial_x g_n(1-\delta) = 1 \tag{B.7}$$

It is easy to check that such a solution to Eq.(3.1) has the following values of C and D

$$C = \sqrt{2\delta - \delta^2} \frac{Q_1^n(1-\delta)}{Q_1^n(1-\delta)\partial_x P_1^{-n}(1-\delta) - P_1^{-n}(1-\delta)\partial_x Q_1^n(1-\delta)} \tag{B.8}$$

$$D = -\sqrt{2\delta - \delta^2} \frac{P_1^{-n}(1-\delta)}{Q_1^n(1-\delta)\partial_x P_1^{-n}(1-\delta) - P_1^{-n}(1-\delta)\partial_x Q_1^n(1-\delta)}$$

The prefactor is then given by

$$\frac{1}{\sqrt{2\pi g_n(-1+\delta)}} = \frac{1}{\sqrt{4\pi}}\sqrt{\left(\frac{\partial_x Q_1^n(1-\delta)}{Q_1^n(1-\delta)} - \frac{\partial_x P_1^{-n}(1-\delta)}{P_1^{-n}(1-\delta)}\right)} \tag{B.9}$$

$$= \frac{1}{\sqrt{4\pi}}\frac{\sqrt{Q_1^n(1-\delta)P_1^n(1-\delta)(2\delta-\delta^2)}}{2^n}\sqrt{\frac{\Gamma\left(\frac{2-n}{2}\right)\Gamma\left(\frac{3-n}{2}\right)}{\Gamma\left(\frac{2+n}{2}\right)\Gamma\left(\frac{3+n}{2}\right)}}$$

C. Path Integral of an Oscillator with variable mass and variable frequency

Here we review some basic properties of path integral and apply them to the evaluation of the prefactor in Eq.(4.7). To be concrete, we shall follow Ref [34]. Consider an oscillator with mass $m(t)$ and frequency $\omega(t)$. What we have in mind, in particular, is a case like Eq.(4.2) which describes the path integral for a tensor mode (in Euclidean spacetime). Comparing Eq.(4.2) with this appendix, the time dependent mass would be $a(t)^3$ and the time dependent frequency would be $(n^2 - 1)/a^2$. We would like to evaluate the propagator $K(x_f, T|x_i, 0)$. The action is given by

$$S = \int_o^T dt \left(\tfrac{1}{2}m \left(\tfrac{dX}{dt}\right)^2 - \tfrac{1}{2}m\omega(t)^2 X^2 \right) \tag{C.1}$$

The problem can be simplified by introducing a new "time" variable u so as to map the present problem to that of an oscillator with unit mass and variable frequency:

$$du = \frac{dt}{m(t)} \tag{C.2}$$

In terms of u the action becomes

$$S = \tfrac{1}{2} \int_{u_i}^{u_f} du \left(\left(\tfrac{dX}{du}\right)^2 - \Omega(u)^2 X^2 \right) \tag{C.3}$$

that is, the action for an oscillator with unit mass and variable frequency $\Omega(u)$. As this is quadratic in $X(u)$, we can expand around a classical solution, $X = x_{cl} + x$, where the classical solution $x_{cl}(u)$ satisfies the equation of motion

$$\frac{d^2 x_{cl}(u)}{du^2} + \Omega(u)^2 x_{cl}(u) = 0 \tag{C.4}$$

where $\Omega(u)$ depends on n. For example, for the nth tensor field perturbation, $\Omega(u) = (n^2 - 1)a(u)^4$. One can use the above solution for x_{cl}, where $x_{cl}(u_i) = X(u_i)$ and $x_{cl}(u_f) = X(u_f)$, to calculate the classical action $S[x_{cl}]$. This gives the saddle point value of the path integral. Following the discussion in Sec. 5, the prefactor of the propagator is given by:

$$\int_{x(u_i)=0}^{x(u_f)=0} D[x(t)] \exp \left[\tfrac{i}{2} \int_{u_i}^{u_f} du \left(\left(\tfrac{dx}{du}\right)^2 - \Omega(u)^2 x^2 \right) \right] \tag{C.5}$$

By a further change of variables, we can change the action to a free particle action. To do so, let $g(u)$ be a solution of the equation of motion (C.4) such that $g(u)$ does not vanish at the initial end point

$$g(u_i) \neq 0 \tag{C.6}$$

implying that $g(u)$ is not a path in Eq.(C.5). Define the following transformation of variables

$$x(u) = g(u) \int_{u_i}^{u} \frac{y'(s)}{g(s)} ds = g(u)F(u) \tag{C.7}$$

where the function $y(u)$ obeys $y(u_i) = 0$. Differentiating Eq.(C.7), one finds the inverse transformation

$$y(u) = x(u) - \int_{u_i}^u \frac{g'(s)}{g(s)} x(s) ds \tag{C.8}$$

In terms of the y variable, we have:

$$\left(\frac{d^2}{du^2} + \Omega(u)^2 \right) x(u) = \left(g''(u) + \Omega(u)^2 g(u) \right) \int_{u_i}^u \left[\frac{y'(s)}{g(s)} \right] ds \tag{C.9}$$
$$+ \frac{g'(u)y'(u)}{g(u)} + y''(u) = \frac{g'(u)y'(u)}{g(u)} + y''(u)$$

so the action becomes

$$S[x(u)] = -\frac{1}{2} \int_{u_i}^{u_f} du \left[F(u) g'(u) y'(u) + F(u) g(u) y''(u) \right] \tag{C.10}$$

Keeping in mind that $x(u)$ vanishes at the boundaries (see Eq.(C.5)), a further partial integration then leads to

$$S[x(t)] = \frac{1}{2} \int_{u_i}^{u_f} du \left[\frac{dy}{du} \right]^2 \tag{C.11}$$

This is just a free particle action, with boundary conditions

$$y(u_i) = 0 \quad ; \quad y(u_f) = \int_{u_i}^{u_f} ds \frac{y'(s)}{g(s)} = 0 \tag{C.12}$$

To impose the (non-local) boundary condition $x(u_f) = y(u_f) = 0$, we introduce

$$\delta(x(u_f)) = \frac{1}{2\pi} \int d\alpha \ e^{-i\alpha x(u_f)} \tag{C.13}$$

The path integral can now be written as

$$\int_{x(u_i)=0}^{x(u_f)=0} D[x(u)] \exp\left(iS[x(u)] \right) = \frac{1}{2\pi} \int_{x(u_i)=0}^{x(u_f) \ arbitrary} \int_{-\infty}^{\infty} d\alpha D[y(u)]$$
$$\det\left[\frac{\delta x}{\delta y} \right] . \exp\left[-i\alpha g(u_f) \int_{u_i}^{u_f} ds \frac{y'(s)}{g(s)} \right] . \exp\left[\frac{i}{2} \int_{u_i}^{u_f} du \left(\frac{dy}{du} \right)^2 \right] \tag{C.14}$$

As the transformation between $x(u)$ and $y(u)$ is linear, the Jacobian $\det\left[\frac{\delta x}{\delta y} \right]$ is independent of $y(u)$. To carry out the above path integral, we just have to complete the square by the use of the new variable

$$\gamma(u) = y(u) - \alpha u g(u_f) \int_{u_i}^u \frac{ds}{g(s)} \tag{C.15}$$

The above path integral thus becomes

$$... = \frac{1}{2\pi} \det\left[\frac{\delta x}{\delta y} \right] \int_{-\infty}^{\infty} d\alpha \exp\left[-\frac{i}{2} \alpha^2 g(u_f)^2 \int_{u_i}^{u_f} \frac{du}{g(u)^2} \right] \tag{C.16}$$
$$\int_{\gamma(u_i)=0}^{\gamma(u_f) arbitrary} D[\gamma(u)] \exp\left[\frac{i}{2} \int_{u_i}^{u_f} \left(\frac{d\gamma}{du} \right)^2 \right]$$

It is easy to carry out the α integral. To do the remaining path integral, one just has to notice that it represents the probability amplitude for finding the particle *anywhere* at the time u_f. This probability is obviously 1.

$$\int_{\gamma(u_i)=0}^{\gamma(u_f)} D[\gamma(u)] \exp\left[\frac{i}{2}\int_{u_i}^{u_f}\left(\frac{d\gamma}{du}\right)^2\right] \tag{C.17}$$

$$= \int_{-\infty}^{\infty} dx K(x, u_f|0, u_i) = 1$$

where $\gamma(u_f)$ is arbitrary. Therefore, we have

$$\int_{x(u_i)=0}^{x(u_f)=0} D[x(t)] \exp\left[\frac{i}{2}\int_{u_i}^{u_f} du\left(\left(\frac{dx}{du}\right)^2 - \Omega(u)^2 x^2\right)\right] \tag{C.18}$$

$$= \det\left[\frac{\delta x}{\delta y}\right]\sqrt{\frac{1}{2\pi i g(u_f)^2 \int_{u_i}^{u_f} \frac{ds}{g(s)^2}}}$$

It remains to find the value of the Jacobian. It can be found by a discretization process. The paths $x(u)$ and $g(u)$ can be replaced by the multidimensional points $(x_0, x_1, ..., x_N)$ and $(y_0, y_1, ..., y_N)$ with $x_k = x(u_k)$ and $y_k = y(u_k)$. The linear transformation can then be approximated as

$$y_n = x_n - \frac{T}{N}\sum_{k=1}^{n}\frac{g'(u_k)}{g(u_k)}\frac{x_k + x_{k-1}}{2} \tag{C.19}$$

Then,

$$J_N = \det\left[\frac{\partial y_i}{\partial x_j}\right] = \prod_{k=1}^{N}\left(1 - \frac{1}{2}\frac{g'(u_k)}{g(u_k)\frac{T}{N}}\right) \tag{C.20}$$

Taking the $N \to \infty$ limit, one gets

$$\det\left[\frac{\delta y}{\delta x}\right] = \exp\left[-\frac{1}{2}\int_{u_i}^{u_f} du\frac{g'(u)}{g(u)}\right] = \sqrt{\frac{g(u_i)}{g(u_f)}} \tag{C.21}$$

So the path integral prefactor becomes

$$\int_{x(u_i)=0}^{x(u_f)=0} D[x(t)] \exp\left[\frac{i}{2}\int_{u_i}^{u_f} du\left(\left(\frac{dx}{du}\right)^2 - \Omega(u)^2 x^2\right)\right] \tag{C.22}$$

$$= \Delta\left[\det\left(-\frac{d^2}{du^2} - \Omega(u)^2\right)\right]^{-1/2} = \left(2\pi i g(u_i)g(u_f)\int_{u_i}^{u_f}\frac{ds}{g(s)^2}\right)^{-1/2}$$

where Δ is a normalization factor. One can check the validity of this result for simple cases like free particle and simple harmonic oscillator. Using the result from Eq.(C.22), we can write

$$\frac{\left[\det\left(-\frac{d^2}{du^2} - \Omega(u)^2\right)\right]}{\left[\det\left(-\frac{d^2}{du^2} - U(u)\right)\right]} = \frac{g(u_i)g(u_f)\int_{u_i}^{u_f}\frac{ds}{g(s)^2}}{G(u_i)G(u_f)\int_{u_i}^{u_f}\frac{ds}{G(s)^2}} \tag{C.23}$$

where $G(u)$, analogous to $g(u)$, is a solution of the equation of motion with an arbitrary $U(u)$. And, like $g(u)$, $G(u)$ also does not vanish at u_i. Now, let g^0 denote the unique solution to $\partial_u^2 + \Omega(u)^2 g(u) = 0$ which satisfies the boundary conditions:

$$g^0(u_i) = 0; \quad \frac{d}{du}g^0(u_i) = 1 \tag{C.24}$$

Let g^1 represent the solution that satisfies

$$g^1(u_i) = 1; \quad \frac{d}{du}g^1(u_i) = 0 \tag{C.25}$$

In Eq.(C.23), let

$$g(u) = g^0(u) + \epsilon g^1(u); \quad G(u) = G^0(u) + \epsilon G^1(u) \tag{C.26}$$

The integral $\int_{u_i}^{u_f} \frac{du}{[g(t)]^2}$ diverges due to the vanishing of g^0 at u_i. However, since almost all the contribution comes from an infinitely small neighborhood of u_i (in the limit when $\epsilon \to 0$), it follows that it diverges like

$$\int_{u_i}^{u_f} \frac{du}{[u - u_i]^2} \tag{C.27}$$

Consequently,

$$\overset{\lim}{\epsilon \to 0} \quad \int_{u_i}^{u_f} \frac{du}{g(u)^2} = \int_{u_i}^{u_f} \frac{du}{G(u)^2} \tag{C.28}$$

So Eq.(C.23) becomes

$$\frac{\left[\det\left(-\frac{d^2}{du^2} - \Omega(u)^2\right)\right]}{\left[\det\left(-\frac{d^2}{du^2} - U(u)\right)\right]} = \frac{g^0(u_f)}{G^0(u_f)} \tag{C.29}$$

Taking $U(u) = 0$ (for a free particle), we get

$$\frac{\left[\det\left(-\frac{d^2}{du^2} - \Omega(u)^2\right)\right]}{\left[\det\left(-\frac{d^2}{du^2}\right)\right]} = \frac{g^0(u_f)}{u_f - u_i} \tag{C.30}$$

Now we can calculate the propagator,

$$K(x_f, T|x_i, 0) \equiv K(x_f, u_f|x_i, u_i) = K(0, u_f|0, u_i) \exp\left(iS[x_{cl}]\right)$$

$$= \left(\frac{\det\left(-\frac{d^2}{du^2} - \Omega(u)^2\right)}{\det\left(-\frac{d^2}{du^2}\right)}\right)^{-1/2} K_0(0, u_f|0, u_i) \exp\left(iS[x_{cl}]\right)$$

$$= \sqrt{\frac{1}{2\pi i g^0(u_f)}} \exp\left(iS[x_{cl}]\right) \tag{C.31}$$

One can switch back to the t variable now using Eq.(C.2). $S[x_{cl}]$ is the action for the solution x_{cl} of the classical equation of motion satisfying the boundary conditions $x(u_i) = x_i$ and $x(u_f) = x_f$.

Now we specialize this calculation of the path integral for an oscillator with varying mass and frequency to our case. Eq.(C.4) generally does not lend itself to simple analysis. However, since we expect the higher modes (with large values of n) to be the major contributors to the suppression of the tunnelling, one can use the WKB analysis to find approximate solutions. This is because, firstly, Eq.(C.4) has the form of a Schrodinger equation for an arbitrary potential $\Omega(u)^2$, and, secondly, the variation in $\Omega(u)^2 x_{cl}(u)$ for higher modes is slow enough. Typically, for the nth mode, $\Omega^2 \sim n^2$ and $\dot{\Omega} \sim n$. So $\dot{\Omega} << \Omega^2$. Using the WKB approximation, we get:

$$x_{cl}(u) = \frac{1}{\sqrt{\Omega}} \exp \left(\pm i \int_{u_i}^{u_f} du \Omega(u) \right) \tag{C.32}$$

Note that in the Euclidean case, $u \to -iu$ (or, equivalently, $t \to -i\tau$). This is what we have done in Eq.(4.10, 4.11) to get the solution for the tensor mode $t_{cl}(\tau)$. Using these classical solutions we have calculated the classical action in Eq.(4.15). As explained in Eq.(C.31), the prefactor is proportional to $(2\pi i g^0(u_f))^{-1/2}$ where $g^0(u)$ satisfies the boundary conditions given in Eq.(C.24). Following Eq.(4.11) it is clear that for tensor modes, the two linearly independent solutions to the classical equation of motion are $\frac{1}{\sqrt{\omega_n}} \exp \left(\pm \int^u du' \omega_n(u') \right)$. And the following linear combination satisfies the required boundary conditions

$$g^0(u_f) = \frac{1}{\sqrt{2\omega_n(u_i)\omega_n(u_f)}} \left(\exp(D_n) - \exp(-D_n) \right). \tag{C.33}$$

as this choice of $g^0(u)$ vanishes at $u = u_i$ where $D_n = 0$. $\frac{dg^0(u)}{du} = 1$ at $u = u_i$. This yields the result in Eq.(4.16).

References

[1] A. Vilenkin, *Creation of universes from nothing, Phys. Lett.* **B117** (1982) 25; *The birth of inflationary universes, Phys. Rev.* **D27** (1983) 2848.

[2] J. B. Hartle and S. W. Hawking, *Wave function of the universe, Phys. Rev.* **D28** (1983) 2960;
S. W. Hawking, *The Quantum State of the Universe, Nucl. Phys.* **B239** (1984) 257.

[3] H. Firouzjahi, S. Sarangi and S.-H. H. Tye, *Spontaneous creation of inflationary universes and the cosmic landscape*, JHEP **0409**, 060 (2004), hep-th/0406107.

[4] A. D. Linde, *Quantum creation of an open inflationary universe, Phys. Rev. D* **58**, 083514 (1998), gr-qc/9802038;
A. Vilenkin, *The quantum cosmology debate*, gr-qc/9812027.

[5] G. W. Gibbons, S. W. Hawking, and M. J. Perry, *Path integrals and the indefiniteness of the gravitational action, Nucl. Phys.* **B138** (1978) 141;
G. W. Gibbons and M. J. Perry, *Quantizing Gravitational Instantons*, Nucl. Phys. B **146**, 90 (1978).

[6] J. D. Brown and C. Teitelboim, *Dynamical Neutralization Of The Cosmological Constant*, Phys. Lett. B **195**,(1987) 177;
R. Bousso and J. Polchinski, *Quantization of four-form fluxes and dynamical neutralization of the cosmological constant*, JHEP **0006**, 006 (2000), hep-th/0004134.

[7] S. W. Hawking, *The Cosmological Constant is Probably Zero*, Phys. Lett. **B134** (1984) 403;
S. R. Coleman, *Why There Is Nothing Rather Than Something: A Theory Of The Cosmological Constant*, Nucl. Phys. B **310**, 643 (1988);
S. Weinberg, *The Cosmological Constant Problem*, Rev. Mod. Phys. **61**, (1989) 1;
S. B. Giddings and A. Strominger, *Axion Induced Topology Change In Quantum Gravity And String Theory*, Nucl. Phys. B **306**, 890 (1988);
I. R. Klebanov, L. Susskind and T. Banks, *Wormholes And The Cosmological Constant*, *Nucl. Phys. B* **317**, (1989) 665;
W. Fischler, D. Morgan, and J. Polchinski, *Quantum nucleation of false vacuum bubbles*, *Phys. Rev.* **D41** (1990) 2638.

[8] A. D. Linde, *Quantum creation of the inflationary universe*, *Nuovo Cim. Lett.* **39** (1984) 401;
A. Vilenkin, *Quantum Creation Of Universes*, Phys. Rev. D **30**, 509 (1984).

[9] V. A. Rubakov, *Quantum Mechanics In The Tunneling Universe*, Phys. Lett. B **148**, 280 (1984);
G. V. Lavrelashvili, V. A. Rubakov and P. G. Tinyakov, *Tunneling Transitions With Gravitation: Breaking Of The Quasiclassical Approximation*, Phys. Lett. B **161**, 280 (1985).

[10] S. W. Hawking, Commun. Math. Phys. **55**, 133 (1977).

[11] G. W. Gibbons and M. J. Perry, Nucl. Phys. B **146**, 90 (1978).

[12] H. Ooguri, C. Vafa and E. Verlinde, *Hartle-Hawking wave-function for flux compactifications*, hep-th/0502211.

[13] S. R. Coleman, *The Fate Of The False Vacuum. 1. Semiclassical Theory*, Phys. Rev. D **15**, 2929 (1977) [Erratum-ibid. D **16**, 1248 (1977)].

[14] J. P. Sethna, *Decay rates of tunneling centers coupled to phonons : An instanton approach*, *Phys. Rev.* **B25** (1981) 5050;
Phonon coupling in tunneling systems at zero temperature : An instanton approach, *Phys. Rev.* **B24** (1981) 698.

[15] A. O. Caldeira and A. J. Leggett, *Quantum Tunneling In A Dissipative System*, Annals Phys. **149**, 374 (1983).

[16] D. Giulini, E. Joos, C. Kiefer, J. Kupsch, I.-O. Stamatescu and H. D. Zeh, *Decoherence and the Appearance of a Classical World in Quantum Theory* Berlin; New York: Springer, 1996; W. H. Zurek, *Decoherence and the transition from quantum to classical*, Physics Today, **44**, *10*, 36 (1991); M. Gell-mann and J.B. Hartle, *Classical Equations for Quantum Systems*, Phys. Rev. D **47**, 3345 (1993).

[17] A. H. Guth, *The inflationary universe: A possible solution to the horizon and flatness problems*, *Phys. Rev.* **D23** (1981) 347;
A. D. Linde, *A new inflationary universe scenario: A possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems*, Phys. Lett. **B108** (1982) 389;

A. Albrecht and P. J. Steinhardt, *Cosmology for grand unified theories with radiatively induced symmetry breaking*, Phys. Rev. Lett. **48** (1982) 1220.

[18] G. R. Dvali and S.-H. H. Tye, *Brane inflation*, Phys. Lett. **B450** (1999) 72, hep-ph/9812483.

[19] S. Kachru, R. Kallosh, A. Linde, J. Maldacena, L. McAllister, and S. P. Trivedi, *Towards inflation in string theory*, JCAP **0310** (2003) 013, hep-th/0308055.

[20] J. J. Halliwell and S. W. Hawking, *The origin of structure in the universe*, Phys. Rev. **D31** (1985) 1777.

[21] C. Kiefer, *Continuous measurement of minisuperspace variables by higher multipoles*, Class. Quant. Grav. **4** (1987) 1369.

[22] C. Kiefer, *Continuous measurement of intrinsic time by fermions*, Class. Quant. Grav. **6** (1989) 561.

[23] J. J. Halliwell, *Decoherence in quantum cosmology*, Phys. Rev. **D39** (1989) 2912.

[24] C. Kiefer, *Decoherence in quantum electrodynamics and quantum gravity*, Phys. Rev. **D46** (1992) 1658;
A. O. Barvinsky, A. Yu. Kamenshchik, C. Kiefer and I. V. Mishakov, *Decoherence in quantum cosmology at the onset of inflation*, Nucl. Phys. **B 551** (1999) 374-396, gr-qc/9812043

[25] U. H. Gerlach and U. K. Sengupta, *Homogeneous Collapsing Star: Tensor And Vector Harmonics For Matter And Field Asymmetries*, Phys.Rev. **D18** (1978) 1773.

[26] A. O. Barvinsky, *Tunneling geometries. 2. Reduction methods for functional determinants*, Phys. Rev. D **50**, 5115 (1994), gr-qc/9311023.

[27] E. R. Harrison, *Classification of uniform cosmological models*, Mon. Not. R. Astron. Soc. **137** (1967) 69.

[28] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, *The hierarchy problem and new dimensions at a millimeter*, Phys. Lett. B **429**, 263 (1998) hep-ph/9803315.

[29] S. Hsu and A. Zee, *A speculative relation between the cosmological constant and the Planck mass*, hep-th/0406142;
A. Kobakhidze and L. Mersini-Houghton, *Birth of the universe from the landscape of string theory*, hep-th/0410213;
L. Mersini-Houghton, *Can we predict Lambda for the non-SUSY sector of the landscape?*, hep-th/0504026.

[30] S. Kachru, R. Kallosh, A. Linde, and S. P. Trivedi, *De sitter vacua in string theory*, Phys. Rev. **D68** (2003) 046005, hep-th/0301240.

[31] M. R. Douglas, *The statistics of string / m theory vacua*, JHEP **05** (2003) 046, hep-th/0303194.

[32] A. O. Barvinsky, A. Y. Kamenshchik and I. P. Karmazin, *One loop quantum cosmology: Zeta function technique for the Hartle-Hawking wave function of the universe*, Annals Phys. **219**, 201 (1992).
A. O. Barvinsky and A. Yu. Kamenshchik, *Tunnelling Geometries: Analyticity, Unitarity and Instantons in Quantum Cosmology* , Phys. Rev. **D 50** (1994) 5093-5114, gr-qc/9311022.

[33] A. O. Barvinsky, A. Y. Kamenshchik and I. V. Mishakov, *Quantum origin of the early inflationary universe*, Nucl. Phys. B **491**, 387 (1997), gr-qc/9612004.

A. O. Barvinsky, A. Y. Kamenshchik and C. Kiefer, *Origin of the inflationary universe Mod. Phys. Lett.* **A 14**, 1083 (1999), gr-qc/9905098.

[34] B. Felsager, *Geometry, Particles And Fields*, Springer, 1981

[35] G. L. Kane, M. J. Perry and A. N. Zytkow, *An approach to the cosmological constant problem(s)*, Phys. Lett. B **609**, 7 (2005), hep-ph/0408169.